REVIEW ON ENERGY DIFFUSION OF RESONANT ELECTRONS DURING WAVE-PARTICLE INTERACTION

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ABSTRACT

Resonant interaction between particles and waves violates the conservation of the magnetic moment and causes diffusion in energy and pitch angle of the interacting particles. If the waves are sufficiently strong and within the right frequency band to resonate with the bulk of the particles, then the resulting pitch angle scattering and precipitation can be extremely important in modifying the structure of the outer zone plasma, in addition to the possibility of producing high energy particles within their 'forbidden zones'.

INTRODUCTION

Whistler mode waves play an important role in the earth's magnetosphere through hitting of relativistic energetic electrons as well as causing their precipitation into the lower ionosphere. In this case electron motion in a coherent packet of Whistler mode wave propagating along the direction of an ambient magnetic field is studied(V IIB).V is velocity of the particle and B is geomagnetic field. The parametric dependence of the diffusion coefficient D(α),

on ambient plasma density (N, el./cm3), magnitude of wave field (B_f^2) and field strength (B) have been studied by Singh. They have shown that significant pitch angle diffusion occurs for the earth radiation belt electrons with energies from a few keV upto a few MeV.

Herein, the variation of diffusion coefficient D(α), on frequency of the whistler wave(f) as well as wave magnetic field (Bw) are studied.

REVIEW OF LITERATURE

An experiment was conducted on Jan 23-24, 1988 for a nine-hour period between 1705 and 0210 UT. In this experiment transmitted frequencies were in the range 1.9-2.9 kHz centred at 2400 Hz. It was found that initial level, growth rate and saturation level showed temporal variations over 5-15 min and 1-2 h time scales. Bandwidth of received signals remained almost constant at 20 Hz. Assuming gyroresonant interaction responsible for observed wave growth and saturation level, Sonwalkar et al computed longitudinal structures, which were found to be in the range 100-2800 km (for 5-15 min time scale) and 1100-25,000 km (for 1-2 h time scale). Sonwalkar et al concluded that though signals were transmitted at 1.9-2.9 kHz frequencies, the receiver did not record all the frequencies. Though 1.9 kHz waves were received throughout the experiment, other waves had an upper cut-off at f > 2.4 kHz. It was also observed in this experiment that:

(i) Electrons with Pitch angle α>400 played an important role in wave-particle interaction, and wave amplification.

(ii)Longitudinal structure in the equatorial plane was greatly dependent on electron's pitch angle.

Method of computations

It can be shown that total velocity (V) of electrons increases when pitch angle (α) is varied but $V_{||}$ is kept constant as is clear from following equations (1 to 3). $V_{\perp}$ is perpendicular velocity of the interacting electron, C is speed of light, and other symbols are self explanatory.

$$\tan\alpha = V_{\perp}/V_{||} \qquad (1)$$

$$\beta = V(\alpha)/c = \sqrt{(V_{\perp}^2 + V_{||}^2)} \qquad (2)$$

$$\gamma = (1-\beta^2)^{-0.5} = [1-(V_{||}/c)^2 \sec^2\alpha]^{-0.5} \qquad (3)$$

The pitch angle dependent total energy of the electrons, on the basis of eqs.(1)-(3), can be expressed as $E(\alpha) = (\gamma-1)\, m0c2$ (4)

When β<<1, E (α) in keV is calculated from $E(\alpha) = 250\ \beta2$ (5)

m0c2 is considered to have a value of 511 keV. Eqs (3)-(5) give a range of velocity and energy of electrons as a function of pitch angle. Since $V_{||}$ constant, and α is a variable, we will get β, $V_\perp$, V, E as pitch angle depended parameters.

Though frequencies transmitted from Siple station were1.9, 2.4, 2.9 kHz, we do not consider only these frequency values but a total elf/vlf frequency range. The resonant parallel velocity ($V_{||}$) of electrons for these frequencies is computed from the following equations :

$$\omega_H / \gamma - \omega = k_{||}.V_{||} \quad (6)$$

$$k_{||}.c = \omega\mu \quad (7)$$

Here, ω_H is electron gyrofrequency (= 2 πfH, where fH= 837.6 kHz/L3, L being Mcllwain parameter), ω the interacting wave angular frequency (=2π.f, f being frequency in Hz), $k_{||}$, the wave vector parallel to geomagnetic field (B0) line, and μ, the refractive index of the medium. Kennel and Petschek [1966] has derived following expression for pitch angle diffusion coefficient D(α) = (V.$\Delta\alpha$)2/2.Δt (8)

Kennel and Petschek [1966] have shown that

$$\Delta\alpha / \Delta t = \omega_H . B_W / B_0 \quad (9)$$

$$\text{And, } \Delta t = 2/\Delta k.V_{||} \quad (10)$$

But group velocity Vg is written as Vg = $\Delta\omega / \Delta k$ and it is well-known that $Vg/V_{||}$ = 2ω/ωH, thus, D(α) can be written as :

$$D(\alpha) = (V^2/\pi)\omega.\omega_H.\ B^2{}_f/B^2{}_0 \quad (11)$$

B2f is wave spectral density (= B2W/Δf, BW= wave magnetic field amplitude, Δf being band width of the signal). Eq. (11) gives D(α) values in m2rad2/s3, whereas it is generally studied in rad2/s Therefore, we divide Eq. (11) by square of speed of light (as V < c) hence D(a) is expressed as D(a)= β2 /π ω ωH B2 f/B20 (12)

The above equation as shown can be reduced to eq.(13) for relativistic resonant electrons as in that case of whistler mode waves V<<C or β =1 D(α) = 1/π .ω. ω_H .B_f^2/B_0^2 (13)

Geomagnetic field can also be easily computed from the formula given as B0 = 31400 gamma/L3 (14)

Sentman and Goertz [1978] have given following expression for diffusion coefficient, D(α)= 2.ωH2.Bw/[B0.(μ.Δω).[Rn(Rn2-1)1/2] (15)

Where μ is refractive index of the medium and Rn is a dimensionless parameter called normalised resonant energy [i.e. resonant energy, in the unit of m0c2(511 keV),m0 is rest mass of the electron and c is the speed of light].Refractive index of the medium(μ) can be expressed as μ2= fP2/f(fH-f) (16)

fP is plasma frequency and in terms of ambient cold plasma density(N) is given by fP2 (in kHz) = 80.63N(cm-3) (17)

Thus equation (13) can be written as

D(α) = 1/π · ω · ωH · B2f / B20

= 2.ωH2.Bw/[B0.(μ.Δω).{Rn(Rn2-1)1/2}]

From above we can have following equation

f.Bw/B0=fH/[μ.{Rn√(R2n−1)}] (18)

Now Bw can be written as a function of certain parameters

Bw = B0.fH/[f.μ.{Rn√(R2n−1)}] (19)

D(α) = 1/π · ω · ωH · B2f / B20

can be simplified as

D(α) = 1/π · 2πf · 2π fH · B2f/(L3x0.314 x 10-4) 2

Which after some more simplification assumes the following form

D(α) = 1.2739 f(kHz). fH (kHz)· L6 · (10-8) · B2f (pt**2/Hz) (20)

RESEARCH METHODOLOGY

Direct observations of whistler-induced electron precipitation on rockets and satellites along with indirect ground-based observations have also been reported.

Method of calculation

Brice [1964] has given following expression for pitch angle change(α) of an energetic electron during its interaction with whistler mode waves

$\Delta\alpha = 0.5\tan\alpha\,[(\mathrm{cosec}2\alpha)/x \quad -1].\ \Delta E/E$ (1)

Where α is pitch angle of resonant electron, x is normalised frequency (ratio of wave frequency, f, to electron gyro-frequency, f(H) and E is electron kinetic energy. It can easily be shown that for frequency range considered (3 –7 kHz) and considered L (the McIlwain parameter) values. 1 can be neglected in the bracketted term on right side of the above equation. Kennel and Petschek [1966] have shown that Δα can also be expressed as

$\Delta\alpha = \omega H.Bw.\Delta t/Bo$ (2)

In above equation, ωH is angular gyrofrequency of the electron (i.e.ωH = 2 p.fH),Bw is wave's magnetic amplitude,Bo is geomagnetic field and Δt is roughly the time a particle at distance Δk/2 out of resonance changes its phase by 1 radian where k is wave number. Δt can thus be calculated from the relation

$\Delta t = 2/VR.\Delta k$ (3)

VR is resonant velocity of the electron. Following formulae of group(resonant)velocity, VG(R), given by Kennel and Petschek[1966] we can write

$VG/VR = 2x$ (4)

Geomagnetic field B (in units of Tesla) is expressed as

$B = 0.314(10\text{-}4)/L3$ (5)

Group velocity,VG is written as

$VG = \Delta\omega/\Delta k$ (6)

Δω is 2p times of Δf where Δf is band-width of the interacting signal.

Equating eqs.(1) and(2) and taking the help of eqs.(3)-(6) we can easily get the expression for electron's energy release rate as under

$\Delta E/E = 0.146(10\text{-}12).Bw(m\gamma).f2.\sin 2\alpha.L6/\Delta f$ (7)

In above equation, frequency (f) and bandwidth (Δf) are measured in Hz, and as shown wave amplitude in milligamma(or pico Tesla i.e.pT).

CONCLUSION

We can summarise above work as under:

1. For a given interacting signal frequency(f),as refractive decreases Bw decreases. The same is true for a constant L value but different interacting frequencies.
2. For a given L parameter diffusion of electrons increases with frequency i.e. at low frequencies it is low & at high frequencies, the diffusion is high.
3. For a constant interacting signal (frequency remaining constant) diffusion coefficient increases with L parameter i.e. it is low at low latitudes & high at high latitudes.

On 15 October 1999, 'charged particle and plasma wave' detectors on the Polar Satellite observed an enhancement in precipitating energetic electrons coincident with an ELF/VLF electromagnetic emission. For reasonable values of the cold plasma density and the wave intensity at the equatorial plane, the calculated precipitation agrees with measurements for an equatorial interaction region approximately 1000 km in length. Shprits et al. (2007) and Albert (2007) have observed that the developed parallel propagation approximation replaces the integration over wave normal distribution with a closed form expression. But unfortunately this equation is not performing well for electron energy $E \geq 1$ MeV, whereas our D(α) expression does not require any kind of integration. It has its performance over all frequency ranges and needs no integration. Thus our D(α) expression is not only suitable for electron precipitation by wave particle interactions but it can also explain other kind of phenomena such as ELF/VLF bursts, occurrence of auroras and amplification/damping oflow/VLF /ELF waves propagating in Whistler mode.

In the last, it is worthwhile to note that our theoretical D(α) values are in agreement with experimental values reported by Davidson and Walt (1977) who have observed D(α) values in the range of 0.001 to 0.050. Thus our D(α) expression is not only easy to use but correct also.

REFERENCES

[1] Albert JM, Simple approximations of quasilinear diffusion coefficients J.Geophys.Res.,A12202,doi:10.1029/2007JA012551, 112(2007)

[2] Angerami JJ and Thomas JO, The distributions of ions and electronsin the earth'sexosphere, Tech. report No. 3412-3,Stanford Univ.,Stanford, California,1963.

[3] Prasad SS, Night time ionic composition and temperature over Arecibo, J. Geophys. Res., 73(1968)6795.

[4] Shprits Y, Kondrashov D, Chen Y, Thorne R and Ghil M,Re-analysis of relativistic radiation belt electron fluxes using CRRES satellite data, a radial diffusion model, and a Kalman filter, Journal of Geophysical Research, A 12216, doi: 10.1029/2007 JA 012579, (112)2007.

[5] Singh DP, Strong diffusion of resonant electrons by VLF waves, Ind J Radio and Space Phys,20(1991)424.