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REVIEW ON DYNAMIC PROBLEMS OF GENERALIZED THERMOELASTIC MEDIUM WITH DIFFUSION

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INTRODUCTION

One of the most important branches of continuum mechanics is the classical theory of elasticity, which is concerned with the systematic study of the response of elastic bodies to the action of forces which deform it. This response in characterized by the stress and strain distributions inside a body that are developed because of the applied tractions or change in temperature. A body is said to be elastic if it regains its original shape when the forces causing deformation are removed. The elastic property of the material is shared by all substances provided that the deformations do not exceed certain limits determined by the constitutive characteristics of the body. The elastic property is characterized mathematically by certain functional relationships connecting forces and deformations.

An elastic solid that undergoes only an infinitesimal deformation and for which the governing material law is linear is called a linear elastic solid.

REVIEW OF LITERATURE

The classical theory of elasticity serves as an excellent model for studying the mechanical behaviour of a wide variety of solid material and is used extensively in civil, mechanical and aeronautical engineering design. This is the oldest established theory governing the behaviour of deformable solid materials, founded in its present form in the early 19th century. In the theory of linear elasticity, we are concerned with an ideal material governed by Hooke's law (1678), which represents a linear relationship between the stresses and strains. Hooke's law has influenced the scientific thoughts for a considerably long period for the classical linear infinitesimal theory of elasticity and its results agreed with experiments quite well.

During the 150 years period following the discovery of Hooke's law, the growth of the science of elasticity proceeded from a synthesis of solutions of special problems. This gave in the early nineteenth century a fragmentary theory of flexure of beams, an incomplete theory of torsion, the rudiments of the theory of stability of columns, and a few isolated results on bending and vibration of plates.

The first attempt to deduce general equations of equilibrium and vibration of elastic solids was made by Navier on May 14, 1821. This date marks the birth of the mathematical theory of elasticity. Navier deduced a set of three macroscopic differential equations for the components of displacement in the interior of an isotropic elastic solid. Navier also obtained the equilibrium equations on the surface of the solid (the boundary conditions) with the aid of Lagrange's principle of virtual work. Navier's work attracted the attention of Cauchy (1789-1857), who, proceeding from different assumptions, gave a formulation of the linear theory of elasticity that remains virtually unchanged to the present day.

RESEARCH METHODOLOGY

Thermoelasticity

The theory of elasticity was extended to include thermal effects. The theory of thermoelasticity is concerned with the influence of the thermal state of an elastic solid upon the distribution of strain and with the inverse effect, that of deformation upon the thermal state of an elastic medium. Thermoelasticity is the interaction between deformation and thermal fields. Thermoelasticity was stimulated by the various engineering sciences. A remarkable progress in the field of aircraft and machine structure has given rise to numerous problems in which thermal stresses play a role of primary importance. It comprises the heat conduction and stress and strain that arise due to the flow of heat. Thermoelasticity makes it possible to determine the stresses produced by the temperature field and to calculate the temperature distribution due to an action of time dependent forces and heat sources.

The change of body temperature is caused not only by the external and internal heat sources, but also by the process of deformation itself. In classical elasticity, coupling terms in heat conduction equation and inertia terms in elastic equations of motion are neglected. But this is not possible if temperature undergoes a large and sudden change. In thermoelasticity, inertia terms are included in the equation of motion. It is desirable for all elasto-dynamic problems to consider the temperature's dependence of displacements giving rise to coupled thermoelastic equations.

The assumptions that are usually made are:

(i) the deformation is very small,

(ii) the materials behave elastically at all times and in the same manner in all directions,

(iii) the temperature field is determined by taking into considerations, the effect of coupling of temperature and strain fields. Temperature field is always dependent on the deformation.

Theory of uncoupled Thermoelasticity

The theory of thermoelasticity deals with the effect of mechanical and thermal disturbances on an elastic body. In the nineteenth century, Duhamel (1837) and Neumann (1885) introduced the theory of uncoupled thermoelasticity. There are two shortcomings of this theory. First, the fact that the mechanical state of the elastic body has no effect on the temperature is not in accordance with true physical experiments. Second, the heat equation being parabolic predicts an infinite speed of propagation for the temperature, which again contradicts physical observations.

Coupled theory of Thermoelasticity

Biot (1956) formulated the theory of coupled thermoelasticity to overcome the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. In this theory, the equations of elasticity and of heat conduction are coupled. However, this theory shares the defect of the uncoupled theory in that it predicts infinite speeds of propagation for heat waves, i.e., when an elastic solid is subjected to a thermal disturbance, the effect is felt at a location far from the source, instantaneously. Among the works in this theory, Weiner (1957) proved a uniqueness theorem, Nickell and Sackmann (1968) obtained some variational principles and Hetnarski (1961, 1964 a, b) has solved some problems in the form of series of functions and for small times. Other important contributions to the subject are attributable to Nowacki (1964, 1965), Nowacki and Ignaczak (1966) and Boley and Tolins (1962). A good review of the subject is found in the books of Nowacki (1962) and Dhaliwal and Singh (1980).

Diffusion

Diffusion can be defined as a random walk, of an ensemble of particles, from regions of high concentration to regions of lower concentration. There is now a great deal of interest in the study of this phenomenon, due to its many geophysical and industrial applications. In integrated circuit fabrication, diffusion is used to introduce “dopants” in controlled amounts into the semiconductor substrate. In particular, diffusion is used to form the base and emitter in bipolar transistors, form integrated resistors, form the source/drain regions in metal oxide semiconductor (MOS) transistors and dope poly-silicon gates in MOS transistors. Study of the phenomenon of diffusion is used to improve the conditions of oil extractions (seeking ways of more efficiently recovering oil from oil deposits). These days, oil companies are interested in the process of thermodiffusion for more efficient extraction of oil from oil deposits.

Thermoelastic diffusion

Thermodiffusion in an elastic solid is due to coupling of the fields of temperature, mass diffusion and that of strain. Heat and mass exchange with the environment during thermodiffusion in an elastic solid. Nowacki (1974 a, b, c, d) developed the theory of thermoelastic diffusion. In this theory, the coupled thermoelastic model is used. This implies infinite speeds of propagation of thermoelastic waves. Dudziak and Kowalski (1989) also discussed the theory of thermodiffusion for solids. Olesiak and Pyryev (1995) discussed a coupled quasi-stationary problem of thermodiffusion for an elastic cylinder. They studied the influences of cross effects arising from the coupling of fields of temperature, mass diffusion and strain. Due to these cross effects, the thermal excitation results in an additional mass concentration and the mass concentration generates the additional field of temperature.

Sherief *et al.* (2004 b) generalized the theory of thermoelastic diffusion, which allows the finite speeds of propagation for thermoelastic and diffusive waves. The development of generalized theory of thermoelastic diffusion by Sherief *et al.* (2004 b) provides a chance to the study the wave propagation in such an interesting media.

Aouadi (2006 a, b) discussed the thermoelastic-diffusion interactions in an infinitely long solid cylinder subjected to a thermal shock on its surface which is in contact with a permeating substance and also studied a problem of variable electrical and thermal conductivity in the theory of generalized thermoelastic diffusion. A detail account of the plane harmonic generalized thermoelastic diffusive waves in heat conducting solids has been considered by Sharma (2007). Deswal and Choudhary (2008) investigated the disturbances in a homogeneous, isotropic elastic medium with generalized thermoelastic diffusion, when a moving source is acting along one of the coordinate axis on the boundary of the medium.

Inversion of the Laplace transform

We shall now outline the method used to invert the Laplace transforms in the above equations. Let $\bar{g}(s)$ be the L.T. of some function *g(t)*. Following Honig and Hirdes (1984), the Laplace transformed function $\bar{g}(s)$ can be inverted as follow:

$$g(t) = L^{-1}[\bar{g}(s)] = \frac{1}{2\pi i}\int_{v-i\infty}^{v+i\infty} e^{st}\,\bar{g}(s)\,ds, \qquad (2.4.1)$$

where $\bar{g}(s) = L[g(t)] = \int_0^\infty e^{-st}\,g(t)\,dt,$ (2.4.2)

with $s = v+iw$; $v, w \in R$.

$v \in R$ is arbitrary but greater than the real parts of all the singularities of $\bar{g}(s)$.

Now $\bar{g}(s) = \int_0^\infty e^{-st}\,g(t)\,dt = \int_0^\infty e^{-(v+iw)t}\,g(t)\,dt,$

$$= \int_0^\infty e^{-vt}\,g(t)\,[\cos wt - i\sin wt]\,dt,$$

$$= \mathrm{Re}[\bar{g}(v+iw)] + i\,\mathrm{Im}[\bar{g}(v+iw)]. \qquad (2.4.3)$$

Substituting eq. (2.4.3) into eq. (2.4.1) ,we get

$$g(t) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} e^{vt}\left(\cos wt + i\sin wt\right)\left[\mathrm{Re}\,\bar{g}(s) + i\,\mathrm{Im}\,\bar{g}(s)\right] i\,dw,$$

$$= \frac{e^{vt}}{2\pi}\left[\int_{-\infty}^{\infty}\left(\mathrm{Re}\,\bar{g}(s)\cos wt - \mathrm{Im}\,\bar{g}(s)\sin wt\right)dw + i\int_{-\infty}^{\infty}\left(\mathrm{Re}\,\bar{g}(s)\sin wt + \mathrm{Im}\,\bar{g}(s)\cos wt\right)dw\right]$$

(2.4.4)

Combining eqs. (2.4.3) and (2.4.4) ,we obtain

$$g(t) = \frac{e^{vt}}{2\pi}\int_{-\infty}^{\infty}\int_{0}^{\infty} e^{-v\tau} g(\tau)\left(\cos w\tau\cos wt + \sin w\tau\sin wt\right)d\tau\, dw$$

$$+ i\frac{e^{vt}}{2\pi}\int_{-\infty}^{\infty}\int_{0}^{\infty} e^{-v\tau} g(\tau)\left(\cos w\tau\sin wt - \sin w\tau\cos wt\right)d\tau\, dw,$$

$$= \frac{e^{vt}}{2\pi}\left[\int_{-\infty}^{\infty}\int_{0}^{\infty} e^{-v\tau} g(\tau)\left(\cos w(\tau - t) - i\sin w(\tau - t)\right)d\tau\, dw\right]. \qquad (2.4.5)$$

In eq. (2.4.5), $\sin w(\tau - t)$ is an odd function of w; therefore, the second integral is zero and the equation is simplified as

$$g(t) = \frac{e^{vt}}{\pi}\left[\int_{0}^{\infty}\int_{0}^{\infty} e^{-v\tau} g(\tau)\left(\cos w\tau\cos wt + \sin w\tau\sin wt\right)d\tau\, dw\right],$$

$$= \frac{e^{vt}}{\pi}\left[\int_{0}^{\infty}\left(\mathrm{Re}\,\bar{g}(s)\cos wt - \mathrm{Im}\,\bar{g}(s)\sin wt\right)dw\right]. \qquad (2.4.6)$$

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Expanding the function $h(t) = e^{-vt} g(t)$ in a Fourier series in the interval $[0, 2T]$, Durbin (1974) derived the approximate formula

$$g(t) = \frac{e^{vt}}{T}\left[-\frac{1}{2}\mathrm{Re}\,\bar{g}(v) + \sum_{k=0}^{\infty} \mathrm{Re}\left[\bar{g}\left(v + i\frac{k\pi}{T}\right)\right]\cos\left(\frac{k\pi}{T}t\right) \right.$$

$$\left. - \sum_{k=0}^{\infty} \mathrm{Im}\left[\bar{g}\left(v + i\frac{k\pi}{T}\right)\right]\sin\left(\frac{k\pi}{T}t\right)\right] - F_1(v,t,T), \qquad (2.4.7)$$

where $F_1(v,t,T)$ is the discretization error given by

$$F_1(v,t,T) = \sum_{k=1}^{\infty} e^{-2vkT} g(2kT + t). \qquad (2.4.8)$$

As the infinite series in eq. (2.4.7) can only be summed up to a finite number N of terms, a truncation error is introduced in the form of

$$F_A(N,v,t,T) = \frac{e^{vt}}{T}\left[\sum_{k=N+1}^{\infty} \left[\mathrm{Re}\left\{\bar{g}\left(v + i\frac{k\pi}{T}\right)\right\}\cos\left(\frac{k\pi}{T}t\right) - \mathrm{Im}\left\{\bar{g}\left(v + i\frac{k\pi}{T}\right)\right\}\sin\left(\frac{k\pi}{T}t\right)\right]\right].$$

(2.4.9)

Hence the approximate value for $g(t)$ is

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$$g_N(t) = \frac{e^{vt}}{T}\left[-\frac{1}{2}\mathrm{Re}\, g(v) + \sum_{k=0}^{N}\left[\mathrm{Re}\left\{\bar{g}\left(v + i\frac{k\pi}{T}\right)\right\}\cos\left(\frac{k\pi}{T}t\right) - \mathrm{Im}\left\{\bar{g}\left(v + i\frac{k\pi}{T}\right)\right\}\sin\left(\frac{k\pi}{T}t\right)\right]\right].$$

(2.4.10)

It is obvious from eq. (2.4.8) that the discretization error can be made arbitrarily small if the free parameter *υT* is large enough. Unfortunately, the truncation error in eq. (2.4.9) may diverge for large values of *υT*.

Two methods are used to reduce the total error. First, the Korrektur method is used to reduce the discretization error. Next, the ε-algorithm is used to reduce the truncation error and hence to accelerate convergence.

With eq. (2.4.10), eq. (2.4.7) can be written in the form

$$g(t) = g_\infty(t) - e^{-2vT} g_\infty(2T + t) - F_2(v,t,T),$$

where the discretization error $|F_2(v,t,T)| << |F_1(v,t,T)|$. Thus, the approximate value of $g(t)$ becomes

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$$g_{NK}(t) = g_N(t) - e^{-2vT} g_{N'}(2T+t), \qquad (2.4.11)$$

Where N' is an integer less than N. Let

$$c_k = \frac{e^{vt}}{T}\left[\mathrm{Re}\left\{\bar{g}\left(v+i\frac{k\pi}{T}\right)\right\}\cos\left(\frac{k\pi}{T}t\right) - \mathrm{Im}\left\{\bar{g}\left(v+i\frac{k\pi}{T}\right)\right\}\sin\left(\frac{k\pi}{T}t\right)\right]. \qquad (2.4.12)$$

According to eq. (2.4.11), eq. (2.4.10) can be expressed as

$$g_N(t) = \frac{1}{2}c_0 + \sum_{k=1}^{N} C_k. \qquad (2.4.13)$$

Now the $\in$-algorithm is described in the following. Let N be an odd natural number, and let

$$s_m = \sum_{k=1}^{m} C_k,$$

be the sequence of partial sums of eq. (2.4.13), we define the $\in$-sequence by

$$\in_{0,m} = 0, \qquad \in_{1,m} = s_m, \qquad m = 1,2,3,...$$

and

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$$\in_{n+1,m} = \in_{n-1,m+1} + \frac{1}{\in_{n,m+1} - \in_{n,m}}, \qquad n = m = 1,2,3,...$$

It can be shown in Honig and Hirdes (1984) that the sequence $\in_{1,1}$, $\in_{3,1}$, ... $\in_{N,1}$ converges to $g_\infty - \frac{1}{2}c_0$ faster than the sequence of partial sums s_m, $m = 1,2,3,...$ The actual procedure used to invert the Laplace transforms consists of using eq. (2.4.13) together with the $\in$-algorithm. The values of υ and T are chosen according to the criteria outlined in Honig and Hirdes (1984).

REFERENCES

[1] Abd-Alla A.N. and Al-Dawy A. A.S. (2000) the reflection phenomena of SV waves in a generalized thermoelastic medium. *Int. J. Math. Sci.*, 23: 529-546.

[2] Anwar M. and Sherief H.H. (1988 a) State space approach to generalized thermoelasticity. *J. Therm. Stresses*, 11: 353-365.

[3] Aouadi M. (2006 a) A generalized thermoelastic diffusion problem for an infinitely long solid cylinder. *Int. J. Mathematics Mathematical Sci.*, 2006: 123-137.

(IJRST) 2011, Vol. No. 1, Issue No. I, Apr-June **ISSN:** 2249-0604

[4] Aouadi M. (2006 b) Variable electrical and thermal conductivity in the theory of generalized thermoelastic diffusion. *Zeitschrift fr Angewandte Mathematik and Physik (ZAMP), 57:* 350-366.

[5] Biot M. (1956) Thermoelasticity and irreversible thermo-dynamics. *J. Appl. Phys.*, 27: 240-253.

[6] Boley B. and Tolins I. (1962) Transient coupled thermoelastic boundary value problems in half-space. *J. Appl. Mech.*, 2: 637-646.

[7] Chandrasekharaiah D.S. (1996 a) A uniqueness theorem in the theory of thermoelasticity without energy dissipation. *J. Therm. Stresses*, 19: 267-272.