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ANALYSIS OF QNMS WITH BLOCKING AFTER SERVICE AND MAXIMIZE THE UNCERTAINTY OF OUTCOMES

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ABSTRACT

An important issue for the performance modelling and evaluation of modern high-speed communications networks is traffic congestion due to the limited capacity of the network resources. Such problems give rise to the phenomenon of blocking.

In this paper a new algorithm is proposed, based on the principle of maximum entropy(ME), for the approximate analysis of arbitrary open queuing network models(QNMs) with finite capacity, external Poisson arrivals and single exponential severs under blocking after service(BAS). Remarks on the validation of the ME algorithm and future work are included.

Keywords:*Queuing networks models(QNM), Maximum Entropy(ME), Principles, Blocking after Service (BAS).*

1. INTRODUCTION

The subject of this paper is the performance of communication networks where the server at each node has a finite buffer and customers move with known probability from one node to another. For convenience the node will be numbered so we will refer to node(j) and its buffer as buffer (j). If a customer, C, wishes to progress from node(k) to node(j) and buffer(j) is full then some strategy must be in place to cater C's next move.

Various approaches are available, one of which is *blocking after service(BAS)* or *transfer blocking*. Under this scheme the blocked customer C must wait at server(k) until buffer(j) is empty. Complications can arise when there are other customers from other upstream are already blocked. The entry to the blocking node is handled on a *first-come-first-enter(FCFE)* basis. Clearly it is possible that the traffic flow in such a system could deteriorate.

The system under consideration here is assumed to have customer inter-arrival rates and nodal service rates which are exponentially distributed and customers are of single class.

Section 2 introduces the M/M/1/N+R queuing delay model as an effective building block in the solution process. The joint ME solution and its computational implementation are presented in Sections 3 and 4. The estimation of the effective service time is carried out in Section 5. The outline of the ME algorithm is given in Section 6. Conclusions and further remarks follow in Section 7.

2. THE M/M/1/N+R DELAY BUILDING BLOCK

The service time for a blocked customer will be increased due to the delay caused by other waiting customers who have arrived earlier from other nodes as well as the last customer in the actual buffer. The delay caused at each node will be estimated and will be used as a building block for the effective service time at its upstream nodes.

Consider a M/M/1/N + R queue with finite capacity, N+R, a single server, R distinct job classes, exponential inter-arrival and service times with a FCFS scheduling discipline. The inter-arrival times of the classes may be different, but the service times are assumed to have the same distribution. There is no restriction on allocation of buffer space except for the last R spaces; these positions can be occupied only when the first N positions are taken and each of the positions is reserved for one and only one class. We will refer to these R positions as the *overflow buffer*. Such an arrangement can be used to model a node in a queuing network under a BAS system.

With reference to the M/M/1/N+R queue at equilibrium, let at any given time $(\mathbf{k},\boldsymbol{\delta}) = (k_1, k_2, ..., k_R, \delta_1, \delta_2, ..., \delta_R)$ be a system state here k_i is the number of jobs of class *i*, $i = 1,2,...,R$ in the shared buffer and $\delta_i, 1 = 1,2,...,R$ is the number of jobs in the overflow buffer of class *i*. Here $\delta_i = 0 \text{ or } 1$.

Let S be a system state where jobs are positioned under an ordered arrangement and Q in the set of all feasible states S.

$S = (k_1, k_2, ..., k_R, \delta_1, \delta_2, ..., \delta_R)$ ... (1)

And $k_1 + k_2 + ... + k_R + \delta_1 + ... + \delta_R \le N + R$ ... (2)

$\delta_i = 0 \text{ if } k_1 + ... + k_R < N$ ... (3)

For each state, let $n_i(S)$ be the number of class i jobs present in state $\mathbf{S}$ and $s_i(S)$, $f(S)$ be the auxiliary functions defined by

$$s_i(S) = \begin{cases} 1, & \text{if the job in service is of class } i \\ 0, & \text{otherwise} \end{cases} \quad ... (4)$$

$$f(S) = \begin{cases} 1, \text{if } \sum_{i=1}^{R} n_i(S) = N + R \\ 0, \text{otherwise} \end{cases} \quad .... (5)$$

Further, let Λ_i be the rate of the overall arrival process, μ_i the service rate of the process, and P(S) and $P(k_1, k_2, ..., k_R, \delta_1, ..., \delta_R)$ the stationary state probabilities.

3. THE JOINT ME SOLUTION

Following earlier implementations of the maximal entropy methodology for the approximate analysis of single-class finite capacity queues, it is assumed that the following mean value constraints about the state probabilities $\{P(S)\}_{S \in Q}$ are known to exist

(i) Normalisation $\sum_{S \in Q} P(S) = 1$ ....(6)

(ii) Utilisation $\sum_{S\in Q} s_i(S)P(S)=\upsilon_i\, 0<\nu_i<1 i=1,2,\ldots,R$ ....(7)

(iii) Mean queue length $\sum_{S\in Q} n_i(S)P(S)=L_i, \upsilon_i<L_i<N+1, i=1,2,\ldots,R$. ....(8)

(iv) Full buffer stateprobability $\sum_{S\in Q} f(S)P(S)=\varphi_i, 0<\varphi_i<1$. ....(9)

As well as the above the flow balance condition is assumed

$$\Lambda_i(1-\pi_i)=\mu\upsilon_i \quad i=1,2,\ldots,R \text{ .... (10)}$$

where π_i is the probability that a customer of class i is "blocked", i.e. a customer of class i occupies the class i virtual buffer position.

These constraints (i) - (iv) follow the approach taken in Kouvatsos and Denazis (1993). If further constraints are enforced the closed-form ME solution is no longer attainable. Consequently the efficiency of the iterative queue-by-queue decomposition algorithm for arbitrary linked QNM is severely affected. However, if any of the constraints were removed the resulting ME solution would be less accurate.

The form of the state probability distribution P(S), $S\in Q$ can be characterised by maximising the entropy functional

$$H(P)=-\sum_{S\in Q} P(S)\log P(S) \text{ .... (11)}$$

subject to the constraints (i) - (iv). The method of undetermined multipliers of Lagrange leads to the solution

$$P(S)=\frac{1}{z}\prod_{i=1}^{R} g_i^{s_i(S)} x_i^{n_i(S)} y^{f(S)} \text{ .... (12)}$$

$$P(k_1,k_2,\ldots,k_R,\delta_1,\ldots,\delta_R)=Z^{-1}\frac{\left(\sum_{j=1}^{R}k_j-1\right)!}{\prod_{j=1}^{R}k_j!}\left(\prod_{j=1}^{R}x_j^{k_j}\right)\left(\sum_{i=1}^{R}k_i g_i\right)y^{\delta(\sum_{j=1}^{R}k_j)}\prod_{i=1}^{R}x_i^{\delta_i}$$

.... (13)

where

$\delta(\sum_j k_j=1)$ if $\sum_j k_j = N$, or 0 otherwise. .... (14)

The aggregate probability that the shared buffer is full and the overflow buffer is empty is

$$P(N)=Z^{-1}\rho\frac{(1-X)}{1-\rho}X^{N-1}y \text{ .... (15)}$$

where $\rho=\sum_{i=1}^{R}\rho_i$, $\rho_i=\Lambda_i/\mu_i$ and $X=\sum_{i=1}^{R}x_i$ .... (16)

The probability that the number in the overflow buffer is $\tilde{\delta}\equiv(\delta_1,\delta_2,\ldots,\delta_R)$ given by

$$P(\tilde{\delta})=Z^{-1}\rho\frac{(1-X)}{1-\rho}X^{N-1}y\prod_{i=1}^{R}x_i^{\delta_i} \text{ .... (17)}$$

The aggregate state probability *P(n)*, *n* = 1, 2, …, *n* is given by

$$P(n) = Z^{-1}\frac{\rho(1-X)}{1-\rho}X^{n-1} \text{ for } 1 \le n \le N-1 \text{.... (18)}$$

And by normalisation

$$P(0) = P(S_0) = Z^{-1} \text{..... (19)}$$

where

$$Z = \left\{1+\frac{\rho X}{1-\rho}\left[1-X^{N-2}+(1-X)X^{N-2}y\prod_{i=1}^{R}(1+x_i)\right]\right\}^{-1} \text{.... (20)}$$

4. THE LAGRANGIAN COEFFICIENTS {xi}, {gi}, {y}

Asymptotic connections to the infinite capacity queue have been used (see Kouvatsos, 1986)to approximate analytically the Lagrangian coefficients $\{x_i\}$and $\{g_i\}$ for single class G/G/1/N queues by letting $N \to \infty$. Assuming that each of (a) $\rho < 1$ and $X < 1$ and (b)$\{x_i\}$, $\{g_i\}$, $\{y\}$ are invariant to the buffer capacity size *N*, the following relationships hold

$$x_i = \frac{\bar{n}_i - \rho_i}{\bar{n}} \text{,.... (21)}$$

$$g_i = \frac{(1-X)\rho_i}{(1-\rho)x_i} \text{,.... (22)}$$

The flow balance condition (v) leads to

$$y = \frac{1-\rho}{1-X} \text{.... (23)}$$

where $\bar{n} = \sum_{i=1}^{R}\bar{n}_i$ and $\bar{n}_i$ is the marginal queue length of a multi-class M/M/1 queue.

The statistics $\{\bar{n}_i, i = 1,2,...,R\}$ can be determined by (see Kouvatsos and Denazis [3])

$$\bar{n}_i = \rho_i + \frac{\Lambda_i}{1-\rho}\sum_{j=1}^{R}\frac{\rho_j^{\,2}}{\Lambda_j} \text{.... (24)}$$

where $\rho_i = \Lambda_i/\mu$ and $\rho = \sum_{j=1}^{R}\rho_j$ .... (25)

The probability of a customer of class *i* being blocked by server *j*, π_{ij} is given by

$$\pi_{ij} = P(N)x_i\prod_{j=1\, j\ne i}^{R}(1+x_i) \text{.... (26)}$$

We also have $\Lambda_i = (1-\pi_{ij})\lambda_i$ so (13) is a non-linear equation involving the values $\pi_{kj}, 1 \le k \le R$.

Denote by $\tilde{\pi}$ the vector $(\pi_{1j}, \pi_{2j}, ..., \pi_{Rj})$ and G: $\mathbf{R}^R \to \mathbf{R}^R$ the vector function defined by

$$G_i(\tilde{\pi}) = P(N)x_i\prod_{j=1\, j\ne i}^{R}(1+x_i) \text{.... (27)}$$

It is necessary to solve the vector equation

$$G(\tilde{\pi}) = \tilde{\pi} \text{.... (28)}$$

This can be achieved by adopting the Newton-Raphson method.

5. AN ESTIMATE FOR THE EFFECTIVE SERVICE TIME

If we consider now the wider context of the whole network, the individual node i , can be solve separately once the effects of other blocking servers upon the service time of server i has been taken into account.

Let $\bar{d}_{ij}$ denote the mean delay experienced by a completing customer at i which is to proceed to node j then

$$\bar{d}_{ij} = P(N)\left[\frac{1}{\mu_j} + \frac{2}{\mu_j}(\sum_{k \neq i} x_k) + \frac{3}{\mu_j}(\sum_{k \neq m \neq i} x_k x_m) + \ldots + \frac{R}{\mu_j}(\prod_{r \neq i} x_r)\right] \quad .... (29)$$

The mean overall delay, d_i ,will be given by

$$d_i = \sum \alpha_{ij} \bar{d}_{ij} \quad .... (30)$$

where α_{ij} is the transition probability from node$i$ to node j.

The effective service time, σ_i , at node i is taken to be

$$\sigma_i = \frac{1}{\mu_i} + \bar{d}_i \quad .... (31)$$

6. OUTLINE OF ME ALGORITHM

An outline of the ME algorithm is presented below.

Begin

Step1. Input arrival rates end service rates for each node.

Step 2. Calculate the actual flow rates through each node of the network.

Step 3. Using (15) find the blocking probability

Step 4. Calculate the effective service time given by (18).

Step 5. Solve the node as a multi-class M/M/1/N+R(with overflow buffers)

End.

The uniqueness of the solution and convergence of the algorithm cannot be proved directly and will have to be established by rigorous testing.The computational cost for the ME algorithm is of O(M^3) where M^3 is the number of operations for inverting the associated Jacobian matrix.

7. CONCLUSIONS

In this paper a new algorithm is proposed, based on the principle of maximum entropy, for the approximate analysis of arbitrary open QNMs with finite capacity, single exponential servers and external Poisson arrivals under BAS. Validation experiments on the credibility of the ME algorithm against simulation are currently being carried out and will be reported in a subsequent paper.

Furthermore, extensions of the ME algorithm to GE-type network models are being made.

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