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# IS LINEAR GYRO-RESONANT INTERACTION RESPONSIBLE FOR 2HZ. SIDE BANDS?

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## ABSTRACT

*Wave-particle interaction plays an important role in the morphology of ionosphere and magnetosphere and wave propagation. Common view is that Nonlinear cyclotron interaction between resonant electrons, in resonance with transmitted signals in the magnetosphere in 1-30 kHz range is a reason to cause side bands. Side band spacing of 20 Hz (and above) can easily be explained through nonlinear amplification process. Wave magnetic amplitude of 3-5 pT for wave-particle interaction to occur in the magnetosphere the need for non-linear interaction as per the established theory of Inan (1977). It is shown that wave magnetic amplitude (Bw) only of ~2pTisneeded to cause small-such as 2-3 Hz side band spacing, which is below 3 pT suggesting that linear gyro-resonant interaction may also be a cause of small side bands.*

***Key words*** *: Wave-particle interaction, Trapped particles, VLF Emissions*
***Pacs. Nos*** *: 94.20Hn, 94.30.Gm, 52.35.Hr*

## 1. INTRODUCTION

Wave-Particle Interactions (WPI) in the magnetosphere assuming both kinds of magnetic field- static over the time scales of interest, as well as dynamic, has been studied considering both experimental and theoretical methods [Gail et al 1990,Albert,2000,Horne et al.2005]. In order to investigate the effect of dynamic magnetospheric processes on WPI the response of interaction mechanism to time dependent perturbations in the magnetic field is tested using global field compressions [Horne et al,2005].In both cases applicability of quasi-linear (linear) and nonlinear amplification mechanisms has been examined[Dowden ,1971, Dowden et al,1978].Side band generation is one of the phenomena which is a result of WPI, like production of x-rays as well as aurora in the magnetosphere and D & E-region perturbation. Dowden et al [1978] have studied linear as well as non-linear amplification mechanisms using spectrograms/phasograms of 6.6 kHz signals which were transmitted from Anchorage, Alaska, a transportable(TVLF) station comprising of VLF transmitter and balloon-lofted antenna being used . With radiated power of transmitted signals 93watt, receivers, in this case, were located at Dunedin, New Zealand and CampbellIsland,i.e. in the conjugate area. The experiment was done in August-September, 1973. Dowden et al [1978] , Dowden[1971], and Brinca[1972], on the basis of the analysis of linear and nonlinear amplification mechanisms, differentiated them as follows:

1. Linear amplification mechanism is most obvious when long trains of whistlers are observed, sometimes with increasing amplitude in successive hops. Mid-latitude hiss appears to be a consequence of amplification gained over several hops [McPherson, 1974a,b]. Some amplification may be required in all observed whistlers.Measurement using artificial signals indicate magnification of 25dB/hop. Sometimes fairly narrow band (`~1kHz), linear amplification is frequency dependent, and varies considerably over periods of several minutes. However, an essential feature is that the output/received signal is proportional to the input signal.

2. On the other hand, in nonlinear amplification the output amplitude is not related to the input amplitude provided that this is above some threshold, that means the nonlinear amplification needs input wave's magnetic field amplitude (Bw) above some threshold. Inan [1977], and Inan et al[1978] found this limit to be around 3-5 pT. The output amplitude may be very large, but even when it is not, the distinguishing feature of non-linear amplification is that the output amplitude takes a finite time to grow. Sometimes it is the only obvious feature and appears as slowness in response to transmitter pulse or amplitude modulation [Dowden et al, 1978, Inan et al[1978]as well as phase reversal modulation [Koon's et al, 1976]. In typical events the nonlinear nature is found to be more obvious [Dowden, 1971, Dowden et al, 1978].

## 2. SIDE BAND OBSERVATIONS

Many workers have made observations of whistler-mode side band instability in the magnetosphere [Bell and Helliwell, 1971, Park and Chang, 1978, Park, 1981]. Bell and Helliwell[1971] observed regular ~0.5 sec amplitude pulsations of key-down transmissions from NAA( located in Cutler ,Maine) at 14.7 and 17.8 kHz when they analyzed the signal observed at Eights, Antarctica ($75^0$S,$77^0$W), using a 300 Hz wide filter. Likhter et al[1971] reported observations of amplitude fluctuations with 0.1-0.5 sec period on long pulse transmissions from a Russian transmitter. Park and Chang[1978] showed examples of sideband generation recorded at Roberval (Canada, $48^0$N, $73^0$W).These sideband were generated by VLF signals transmitted into the magnetosphere by the transmitters established at Siple, Antarctica($76^0$S, $84^0$W; L= 4.23, L is McIlwain parameter). It was observed generally that when a monochromatic wave was injected into the magnetosphere, the output wave often contained frequencies different from the transmitted frequency. This fact is clear evidence of the nonlinear character of the magnetospheric wave amplification [Park, 1981]. Inan [1977] and Inan et al [1978] have shown that for non-linear gyro-resonant interaction between coherent signals and energetic electrons the wave magnetic amplitude (Bw) must be greater than/equal to 3-5 pT.

### 2.1 Experimental observation of very small (2-4 Hz) side bands

Dowden [1971] and, Dowden et al[1978], are of the view that VLF transmitter experiments conducted in July,1977 at Siple, Antarctica(L~ 4) injecting long (≥1 s) key-down signals into the magnetosphere generate side bands as a result of interactions of injected whistler mode waves with energetic(resonant ) particles(i.e. a kind of WPI).After a great analysis of spectral characteristics of recorded spectrograms, Park[1981] concluded that:

(1) Side band amplitudes may be symmetrical/ asymmetrical about the carrier, but in asymmetrical case it is usually the upper side band that is stronger

(2) Side band separations tend to remain constant at a certain time. No case was observed showing variation of side band separation with carrier amplitude. The side band amplitude is usually 10 dB or more below the carrier amplitude, but some time it can exceed the carrier amplitude and also trigger emissions

**(**3) The side band frequency spacing varies from very small value of 2 Hz to 100 Hz(pl. see 8th line of abstract, p.2286; 9th line of summary,p.2289), but it bears no simple relationship to the carrier amplitude. In the same manner, side band amplitudes, too, have no relation with the injected signal/carrier frequency. Multiple side bands are often observed, and their frequency separations from the carrier may or may not be harmonically related. Park [1981] studied the reason of side band spacing up to 20 -100 Hz [point-3] (adopting non-linear amplification mechanism) giving no consideration to small 2-4 Hz side band spacings. We try to show that these side bands separations can be explained using linear amplification mechanism.

## 3. METHOD OF COMPUTATION

**S**ide band generation in whistler mode via a non-linear Doppler-shifted cyclotron interaction between energetic electrons and the whistler mode carrier signal has been studied by Ikeda[2002] and derived following formula for side band spacing generated because of WPI for whistler waves($\omega_H > \omega$, $\omega_H$ is angular cyclotron frequency of energetic electrons and $\omega$, the wave angular frequency)

$$F_{spacing} = 0.0633 \quad n.\sqrt{} \quad (k.V_\perp.\Omega \quad (Bw)) \qquad (1)$$

where n is order of side band generation, k is wave number, $V_\perp$ is perpendicular speed of resonant electrons and $\Omega$(Bw) is the angular frequency of wave(trapping) magnetic field(also known as wave gyrofrequency). Wave number k can be calculated from following formula]

$$k = \omega.\mu \quad /c \qquad (2)$$

where c is speed of light and μ is known as refractive index of the medium which can be computed from following equation [Singh and Singh,2006]:

$$\mu = \sqrt{} \quad \{\omega_p^2/\omega \quad (\omega_H-\omega)\} \qquad (3)$$

$\omega_p$ is plasma frequency of electrons. The resonant velocity of electrons ($V_R$) can be calculated from the equation that follows [singh, 1991]

$$V_R = (\omega_H - \omega)/k \tag{4}$$

Ambient plasma density to calculate angular plasma frequency at considered L shell of 4.23 (where the duct between Siple and Roberval was found to be located at) was taken to be 313 el.cm$^{-3}$. This value corresponds to diffusive equilibrium model of Angerami and Thomas [1963] and have been used earlier by Singh [1991, 1992], and Singh et al[1994]. Energetic electrons gyrofrequency $f_H$ (=$\omega_H/2\pi$) can be computed from following equation

$$f_H \text{ (in kHz)} = 873.6/L^3 \tag{5}$$

## 4. RESULTS AND DISCUSSIONS

As the experiment conducted at Siple had two interacting frequencies ($f = \omega/2\pi$) we too, do our calculations at 4.02 and 4.44 kHz. We calculate the values of side band spacing at 4 pT(mean value of 3-5 pT) wave magnetic amplitude to cause non-linear amplification mechanism. Eq.(1) clearly indicates that minimum spacing will be caused by low pitch angles for which $\alpha \geq \alpha_0$, we adopt $8^0$, $12^0$ ,$16^0$and $20^0$ pitch angles.Table-1 shows $F_{spacing}$ at two considered frequency. It is clear from the Table-1 that as pitch angle increases side band spacing increases but decreases as frequency increases. It is clear from the table that we need Bw below 4 pT as all values of side band spacings are higher than 2 Hz .In Table-2 we produce band widths at Bw less than 4pT for amplitudes of 1-3pT at interval of 0.5pT for a pitch angle of 100 .Table-2 depicts that we can get 2-3 Hz spacing at both transmitter frequencies indicating that 2-3 Hz spacings were perhaps due to linear amplification. It also suggests that that we cannot get 2-3 Hz spacing at considered pitch angles of $<10^0$. We also show in Fig.1 that we can get small sideband spacings for Bw<3pT. So for taking Fig.1 in consideration and computing side band spacing, we consider wave amplitude of 1.5 pT and now consider minimum possible pitch angle to cause wave growth(i.e. loss cone pitch angle)$\alpha_0$ i.e. half loss cone pitch angle . As already written Park [1981] has shown that the duct was located in this case at L=4.23. At this location half loss cone pitch angle[Singh,1991,1992] is found to be $5.03^0$ .Table 3 shows data suggesting that 2-3 Hz side bands were generated by Linear mechanism, but to prove our point we make tests for the purpose for 4.04kHz only.

## 5. TEST OF NON-LINEARITY: NUMBER OF TRAP OSCILLATIONS

Inan et al[1978] have utilized this test to discuss non-linear wave-particle interactions between energetic electrons and whistler mode waves taking place in the magnetosphere considering the top of the $60^0$ geomagnetic field line (L = 4) as the region for effective interaction. Cornilleau-Wehrlin and Gendrin[1979] as well as Raghuram et al[1977] have shown that to explain generation of side bands associated with received VLF signals at L=3.8, the wave magnetic amplitude should be of the order of 4pT, and the phenomenon cannot be explained by linear mechanism. In our case this value is below 4 pT and so linear mechanism may be taking place in case of events we are discussing here. But whereas these workers do their calculations at $\alpha = 45^0$, in our case this angle is equatorial half loss cone pitch angle ($\alpha \sim \alpha_0$). Because of this we take the help of the 'method of trap Oscillation' adopted earlier by Dowden [1971], Brinca[1972] and Inan[1977]. Number of trap oscillations (N) in non-linear amplification should have a threshold of 2. As written earlier, Table-3 shows various data used in the calculation of number of trap oscillation, and other important parameters. We find that N can be computed for the data of Table-3 as under:

$$N = \omega_T . L_{int} / (2\pi . V_R) \qquad (6)$$

Where $\omega_T$ is trap oscillation frequency(in rad/s) and $L_{int}$ is the trapping length which is a function of wave magnetic amplitude(Bw), pitch angle of the trapped electron($\alpha$), and resonant velocity($V_R$). Trap oscillation frequency ($\omega_T$) and trapping length ($L_{int}$) can be expressed as:

$$\omega_T = \sqrt{\{k . V_\perp . \Omega(Bw)\}} \qquad (7)$$

$$L_{int} = 2.\Omega \ (Bw). \ (LRo)^2 \ .\tan\alpha.\{3/(1-x) \ +\tan^2\alpha\}/9.V_R \qquad (8)$$

In Eq.(8), Ro is radius of earth(6370 km) and x is normalized frequency. We see that for the transmitted frequency of 4.02 kHz, Bw =1.5 pT and pitch angle of $5.03^0$

$$N = (34.64).(45830)/ \{(6.28) . \ (1.94(10^7))\} = 0.013$$

For the above values but at transmitted frequency of 4.44 kHz we obtain the N value to be 0.016. These values are less than minimum required value of 2 suggesting that the process taking place in generation of 2-3 Hz side band spacing corresponds to linear amplification.

**Table 1- $F_{spacing}$ (Hz) computed at three different pitch angles( $8^0$,$12^0$ ,$16^0$,$20^0$ ) and interacting signals of 4.02 and 4.44 kHz. The wave magnetic amplitude is taken to be 4 pT.**

| f | 4.02 kHz | f =4.44 kHz |
|---|---|---|
| $8^0$ | 4.32 | 4.20 |
| $12^0$ | 5.32 | 5.17 |
| $16^0$ | 6.18 | 6.00 |
| $20^0$ | 6.96 | 6.76 |

**Table 2- $F_{spacing}$ (Hz) computed at three different pitch angles ( $10^0$ only) and interacting signals of 4.02 and 4.44 kHz. The wave magnetic amplitude is taken to be 1-3 pT at intervals of 0.5pT.**

| | f =4.02 kHz | | | | | f =4.44 kHz | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Bw | 1 | 1.5 | 2 | 2.5 | 3 | 1 | 1.5 | 2 | 2.5 | 3 |
| 10° | 2.41 | 2.95 | 3.41 | 3.81 | 4.17 | 2.35 | 2.88 | 3.34 | 3.72 | 4.07 |

**Table 3 – Various data used in the calculation of number of Trap Oscillation (N)**

L = 4.23
$f_H$ = 11.54 kHz
$L_{int}$ = 45830 m
$\omega_T$ = 34.64 rad/s

n = 313 ele./cc
f = 4.02 kHz

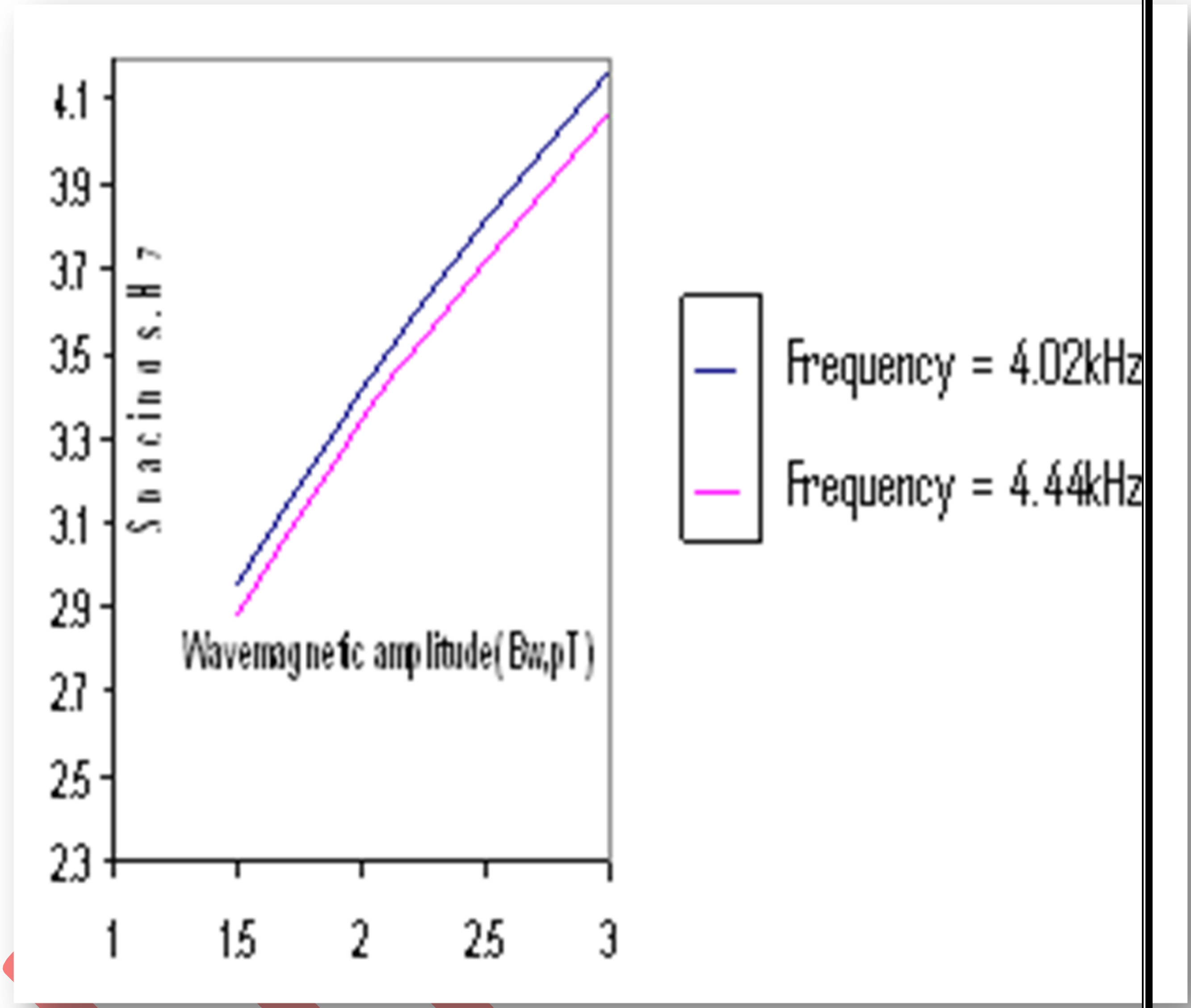

**Fig.1-Variation of sideband spacings (Hz) with wave magnetic amplitude (Bw, in pT).**

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