

An Analysis Of The Mathematical Modelling To Understand And Analyze Complex Systems : Fundamentals Concept

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ABSTRACT

Mathematical modeling is an indispensable instrument for breaking down complex systems, described by unpredictable collaborations and developing ways of behaving. This paper presents an extensive survey of mathematical modeling methods used to comprehend and analyze complex systems. We talk about different methodologies, including deterministic and stochastic models, agent-based models, and network theory. Through itemized contextual analyses in fields like ecology, epidemiology, and engineering, the paper shows how these models are applied to genuine issues. We finish up with a conversation on the difficulties and future headings in mathematical modeling for complex systems.

INTRODUCTION

Complex systems, incorporating many disciplines from natural ecosystems to engineered networks, display ways of behaving and dynamics that are challenging to foresee because of their perplexing part associations and new properties. Mathematical modeling gives an organized way to deal with simulate, analyze, and figure out these systems. This paper audits key methodologies and progressions in mathematical modeling, meaning to clarify their applications.

Mathematical modeling gives a systematic way to deal with understanding and investigating complex systems. By making an interpretation of genuine peculiarities into mathematical structures, models empower researchers to simulate, investigate, and foresee system conduct under shifting conditions. Using differential equations, agent-based models, network analysis, and different strategies, mathematical models offer important bits of knowledge into the dynamics of complex systems, assisting with unwinding the hidden mechanisms driving noticed designs.

As complicated and challenging to comprehend as complex systems might be, many have been justified through science. Mathematical models are utilized to excuse the rationale and association of a system, and analyze their state, as well as foresee future ways of behaving. Most examinations in complex systems are pointed toward making sense of natural peculiarities (like self-association or self-propagation). In the right on time to mid-twentieth century the hybrid of disciplines to be specific in math, biology, physics, and computer science prompted a cooperative exertion in better figuring out system ways of behaving.

Mathematical models are utilized to formalize complex hierarchical principles and ways of behaving by abstracting basic rule-sets from which higher request complexity and association arise. Such models can then be utilized to foresee the condition of a system and its way of behaving at a given time from now on (Rubinow 1975). Complex systems are separated into more modest parts and simulated as reductionist models, zeroing in on a specific part or part of the system. As the model advances, more subtleties are added to make it as firm as conceivable in addressing certifiable situations. Subsequently, countless mathematical models utilized in the utilization of complex issues have been propelled by natural models. The models are in many cases dynamic — that is, models whose state change with time

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in light of vacillations in requirements, data and climate (Kar 2016). Subsequently, models are reductive portrayals of true peculiarities; their objective isn't to in a real sense reproduce a peculiarity, yet to make sense of it through straightforward logistics that can later be applied to comprehend and foresee future ways of behaving of the system(s).

TYPES OF MATHEMATICAL MODELS

Mathematical modeling is the cycle by which this present reality circumstance is addressed and deciphered by the utilization of unique images. The course of mathematical modeling is viewed as a workmanship as well as science. The science perspective arrangements with the points expected to execute the important stages in the modeling system. It is notable that the issues of the truth are rarely same; subsequently highlights, for example, imagination, instinct and prescience likewise contribute critically in the modeling system. These highlights comprise the craftsmanship part of mathematical modeling and make it exceptionally requesting as well as trying movement. The science viewpoint can be valued in a detached learning mode yet the craftsmanship perspective can be valued simply by learning in a functioning mode that is, by building mathematical models and gaining as a matter of fact. The specialty of modeling is viewed as vital during iterative phase. There are two distinct kinds of mathematical models, 1) Deterministic Models and 2) Stochastic Models

1. Deterministic Models-

The essential models partition the relating populaces into defenseless, irresistible and recuperated (or eliminated). These are named as SIR models for various diseases. The extents of people in each class of such models are given by differential equations. A model for development of infection was built (Pokhariyal, 1986) by taking into account the accompanying elements: The design and opposition (invulnerability) given by the tissues. The climate and other outside factors (counting the other contending microbes) supporting or restricting the infection. The descendants and their degree of endurance The development of infection whenever (from beginning infection to the level-off time) was given as

$$\frac{dp_t(I)}{dt} = \gamma \{1 - p_t(I)\} \{p_t(I) - p_0(I)\} \dots(1)$$

where can be named as a parameter. The unequivocal arrangement of (1), disagrees with the boundary conditions (the genuine circumstance), in this way the verifiable articulation for infection extent p (I) was found as:

$$p_t(I) = L - \{L - p_0(I)\} \exp \left[- \int_0^t \gamma \{p_t(I) - p_0(I)\} dt \right] \dots\dots(2)$$

At the time for the highest infection proportion growth $t = t_c$ it was found that

$$f_{t_c}(I) = \gamma \left[\frac{L - p_0(I)}{2} \right]^2 \dots\dots\dots(3)$$

The preventive estimates under various conditions were proposed. The mathematical reason for creating and surveying simulated disease profiles in plant microbe pestilences with accentuation on exactness was introduced (Pokhariyal and Rodrigues, 1993). A computational model was created and through reproduction decision about its predominance over different models was laid out. It was shown that the result of model parameters is a steady as:

$$\gamma t_c \{L - p_0(I)\} = 2 \ln 2 = 1.3862 \dots\dots\dots(4)$$

A relationship between model parameters was shown as:

$$\gamma m t_c^2 = 0.48045 \dots\dots\dots(5)$$

The equation (5) was used for recommending viable harvest advancement systems. A deterministic model for HIV infection and its application was created (Pokhariyal and Simwa, 2004) by taking into account the different phases of infection through relating differential equations and the boundary conditions, which are connected with the CD4 cell include in the patient's body. Utilizing the information from patients' records various situations can be simulated for the advancement of HIV/Helps antiretroviral drugs treatment system.

2. Stochastic Models -

A computable depiction of a genuing peculiarity is known about as a mathematical model of that peculiarity. Examples proliferate, from the basic equation $S = \frac{1}{2}gt^2$ portraying the distance S went in time t by a falling item beginning very still to a complex computer program that simulates a natural populace or an enormous modern system.

In the last analysis, a model is passed judgment on utilizing a solitary, very down to earth, element, the model's convenience. Some models are valuable as accurate computable remedies of conduct, concerning example, a stock model that is utilized to choose the complete number of elements to stock. One more model in an alternate situation might provide just wide individual information about the relationships between and relative importance of a few component impacting an occasion. Such a model is helpful in a comparably significant yet very unique manner. Examples of assorted kinds of stochastic models are spread all through this book.

Such frequently referenced credits as authenticity, style, legitimacy, and reproducibility are significant in assessing a model just to the extent that they bear on that model's definitive handiness. For example, it is both unrealistic and very inelegant to see the rambling city of Los Angeles as a geo-metrical point, a mathematical object of no size or aspect. However it is very valuable to do precisely that while utilizing round calculation to determine a base distance extraordinary circle air course from New York City, another "point."

There is no such thing as the best model for a given peculiarity. The down to earth measure of helpfulness frequently permits the presence of at least two models for a similar occasion, however filling unmistakable needs. Think about light. The wave structure model, in which light is seen as a persistent stream, is completely satisfactory for designing eyeglass and telescope focal points. In contrast, for understanding the effect of light on the retina of the eye, the photon model, which perspectives light as little discrete firecrackers, is liked. Neither one of the models supplants the other; both are pertinent and helpful.

The modern way to deal with stochastic modeling is in a comparable soul. Nature doesn't direct a one of a kind meaning of "probability," similarly that there is no nature-forced meaning of "point" in math. "Probability" and "point" are terms in unadulterated math, characterized exclusively through the properties put resources into them by their separate arrangements of aphorisms. There are, however, three general principles that are in many cases valuable in relating or connecting the theoretical components of mathematical probability theory to a genuine or natural peculiarity that will be modeled. These are (I) the rule of similarly possible results, (ii) the guideline of long run relative recurrence, and (iii) the standard of chances making or emotional probabilities. Historically, these three ideas emerged out of generally fruitless endeavors to characterize probability regarding actual encounters. Today, they are relevant as rules for the task of probability values in a model, and for the translation of the decisions of a model as far as the phenomenon under study.

The advanced way to deal with stochastic modeling is to separate from the meaning of probability from a specific sort of use. Probability theory is a proverbial construction), a piece of unadulterated science. Its utilization in modeling stochastic peculiarities is essential for the more extensive domain of science and matches the utilization of different parts of science in modeling deterministic peculiarities.

To be valuable, a stochastic model should mirror that large number of parts of the peculiarity under concentrate on that are pertinent to the current inquiry. In addition, the model should be agreeable to estimation and should permit the deduction of significant expectations or suggestions about the peculiarity.

METHODOLOGIES IN MATHEMATICAL MODELING

1. Agent-Based Models-

Finally, we have arrived at the exceptionally last part, on agent-based models (ABMs). ABMs are apparently the most summed up structure for modeling and recreation of complex systems, which really incorporate both cellular automata and dynamical networks as extraordinary cases. ABMs are generally utilized in various disciplines to simulate dynamical ways of behaving of systems made of countless elements, like brokers' ways of behaving in a market (in economics), movement of individuals (in social sciences), connection among representatives and their exhibition improvement (in hierarchical science), rushing/tutoring conduct of birds/fish (in conduct ecology), cell development and morphogenesis (in formative biology), and aggregate way of behaving of granular materials (in physics). Figure 2 shows a schematic representation of an ABM. It is somewhat difficult to characterize definitively what an agent-based model is, on the grounds that its modeling suppositions are completely open, and hence there aren't numerous key requirements that describe ABMs. Yet, this is the very thing that I desire to be a moderate meaning of them: There are a couple of catchphrases in this definition that are vital to grasp ABMs.

The principal watchword is "computational." ABMs are normally executed as recreation models in a computer, where every agent's social principles are depicted in an algorithmic design as opposed to a simply mathematical way. This permits modelers to execute complex inward properties of agents and their nontrivial social standards. Such portrayals of complex individual characteristics are exceptionally esteemed particularly in social, hierarchical and the executives sciences, where the modelers need to catch complex, practical ways of behaving of human people. For this reason ABMs are especially famous in those exploration regions.

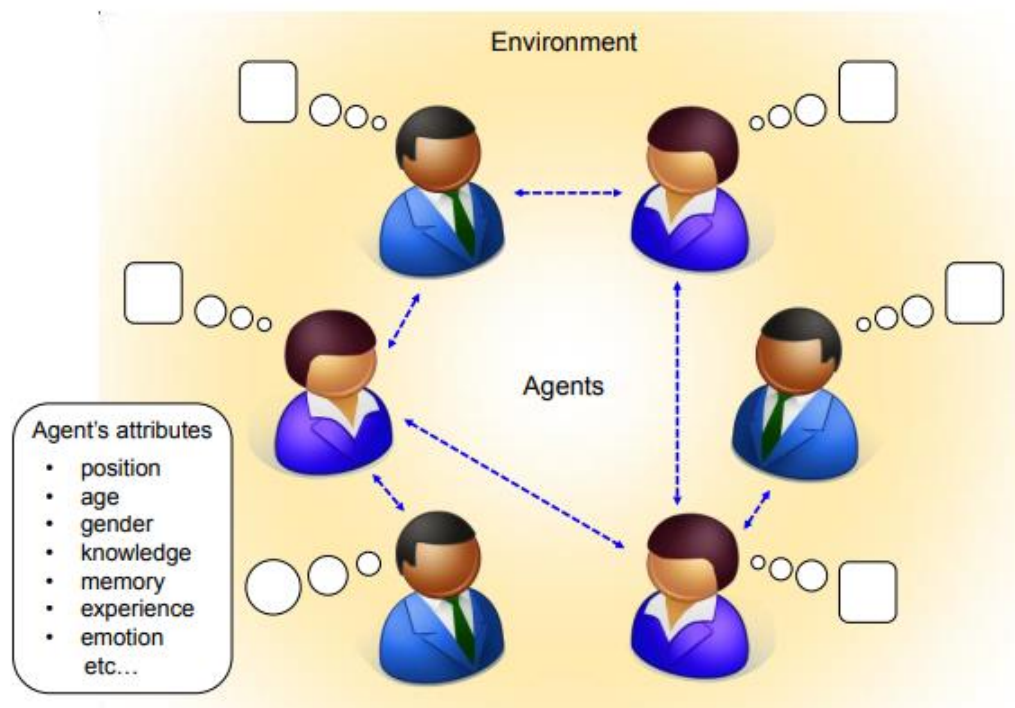


Figure 2: Schematic illustration of what an agent-based model (ABM) looks like.

This, obviously, comes at the expense of logical manageability. Since agents can have quite a few complex properties and social principles, it is for the most part difficult to direct a rich mathematical analysis of an ABM (which is the reason there is no "Analysis" section on ABMs after this one). Consequently, the analysis of ABMs and their reenactment results are generally done utilizing more traditional factual analysis usually utilized in social sciences, e.g., by running Monte Carlo reproductions to get appropriations of result estimations under multiple experimental conditions, and afterward leading measurable speculation testing to check whether there was any tremendous contrast between the different experimental conditions. In this sense, ABMs could act as a virtual swap of experimental fields for researchers.

The second catchphrase in the definition above is "many." In spite of the fact that it is in fact conceivable to make an ABM made of only a couple of agents, there would be little requirement for such a model, on the grounds that the commonplace setting in which an ABM is required is when researchers need to concentrate on the aggregate way of behaving of an enormous number of agents (any other way it would be adequate to utilize a more regular equation-based model with few factors). Thusly, run of the mill ABMs contain a populace of agents, very much like cells in CA or hubs in dynamical networks, and their dynamical ways of behaving are concentrated on utilizing computational recreations.

The third catchphrase is "discrete." While there are a few ambiguities about how to thoroughly characterize an agent, what is regularly acknowledged is that an agent ought to be a discrete individual element, which has an unmistakable boundary among self and the outside. CA and network models are made of discrete parts, so they qualify as exceptional instances of ABMs. Meanwhile, constant field models take on persistent spatial capabilities as a portrayal of the system's state, so they are not viewed as ABMs. There are sure properties that are by and large expected in agents and ABMs, which on the whole characterize the "agent-ness" of the substances in a model. Here is a rundown of such properties:

Typical properties generally assumed in agents and ABMs

- Agents are discrete entities.
- Agents may have internal states.
- Agents may be spatially localized.
- Agents may perceive and interact with the environment.
- Agents may behave based on predefined rules.
- Agents may be able to learn and adapt.
- Agents may interact with other agents.
- ABMs often lack central supervisors/controllers.
- ABMs may produce nontrivial "collective behavior" as a whole.

Note that these are not severe prerequisites for ABMs (maybe with the exception of the first). A few ABMs don't have inside conditions of agents; some don't have space or climate; and some have focal regulators as unique sorts of agents. Subsequently, which model properties ought to be integrated into an ABM is truly up to the goal of your model. Before we continue on toward genuine ABM building, I might want to point out that there are a couple of things we should be especially cautious about when we construct ABMs. One is tied in with coding. Executing an ABM is normally considerably more coding-extreme than carrying out other less difficult models, incompletely in light of the fact that the ABM structure is so unassuming. The way that there aren't numerous requirements on ABMs additionally implies that you need to deal with every one of the subtleties of the reenactment yourself. This naturally builds how much coding you should do. Furthermore, the more you code, the almost certain an unforeseen bug or two will slip into your code. It is in this manner vital to keep your code basic and coordinated, and to involve the prescribed procedures in computer programming (e.g., modularizing sections, adding a lot of remarks, executing systematic tests, and so on),

to limit the dangers of having bugs in your recreation. If conceivable, it is attractive to have multiple individuals test and completely take a look at your code.

Building an Agent-Based Model-How about we begin with agent-based modeling. As a matter of fact, there are numerous incredible instructional exercises currently something else about how to construct an ABM, particularly those by Charles Macal and Michael North, prestigious agent-based modelers at Argonne Public Lab. Macal and North recommend considering the accompanying viewpoints when you design an agent-based model:

1. Explicit issue to be addressed by the ABM 2. Design of agents and their static/dynamic ascribes 3. Design of a climate and the manner in which agents associate with it 4. Design of agents' ways of behaving 5. Design of agents' shared collaborations 6. Accessibility of information 7. Method of model approval Among those points, 1, 6, and 7 are about basic logical methodologies. It is essential to remember that simply constructing an erratic ABM and acquiring results by reproduction wouldn't create any logically significant end. For an ABM to be deductively significant, it must be assembled and utilized in both of the accompanying two reciprocal methodologies:

A. Fabricate an ABM utilizing model suppositions that are gotten from observationally noticed peculiarities, and afterward produce beforehand obscure aggregate ways of behaving by reproduction.

B. Assemble an ABM utilizing speculative model suppositions, and afterward repeat observationally noticed aggregate peculiarities by reenactment.

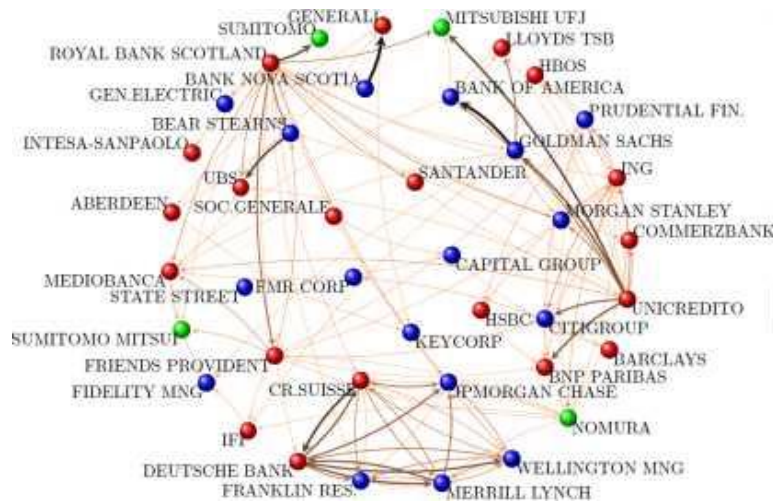
The previous is to utilize ABMs to make expectations utilizing approved hypotheses of agent ways of behaving, while the last option is to investigate and foster new clarifications of experimentally noticed peculiarities. These two methodologies are different concerning the sizes of the known and the obscure (A purposes miniature known to create large scale obscure, while B utilizes microunknown to replicate large scale known), however significantly, one of those scales ought to be grounded on deep rooted exact information. In any case, the recreation results would have no ramifications for this present reality system being modeled. Obviously, a free investigation of different aggregate dynamics by testing speculative agent ways of behaving to create theoretical results is very fun and instructive, with loads of scholarly advantages of its own. My point is that we shouldn't confuse results got from such exploratory ABMs as an approved forecast of the real world.

Meanwhile, things 2, 3, 4, and 5 in Macal and North's rundown above are more centered around the specialized parts of modeling. They can be converted into the accompanying design undertakings in genuine coding utilizing a programming language like Python:

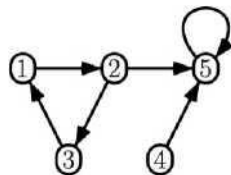
Design tasks you need to do when you implement an ABM

1. Design the data structure to store the attributes of the agents.
2. Design the data structure to store the states of the environment.
3. Describe the rules for how the environment behaves on its own.
4. Describe the rules for how agents interact with the environment.
5. Describe the rules for how agents behave on their own.
6. Describe the rules for how agents interact with each other.

2. Network Theory-

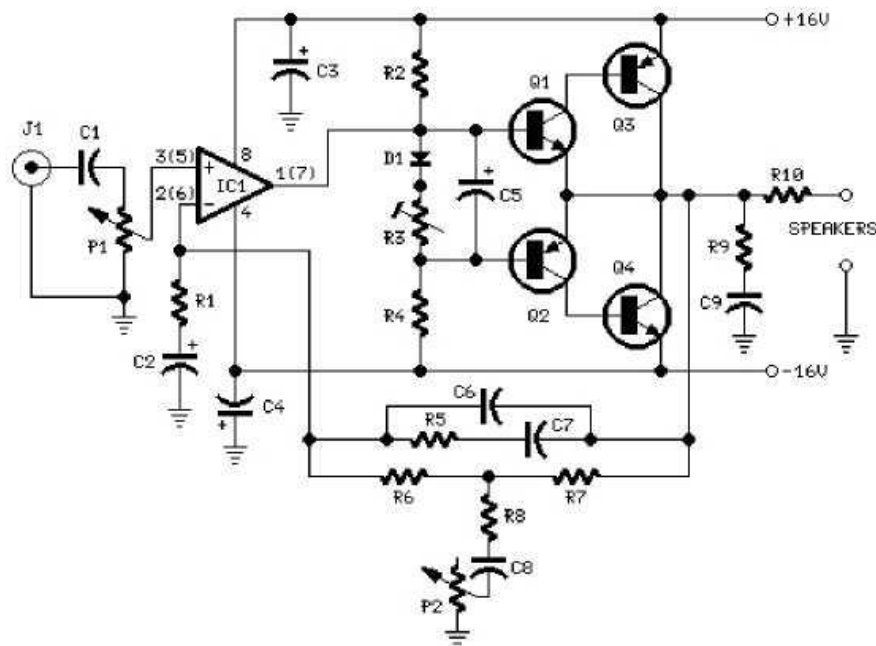


Network theory is the investigation of complex collaborating systems that can be addressed as graphs furnished with additional construction. A graph is a lot of vertices associated by edges:

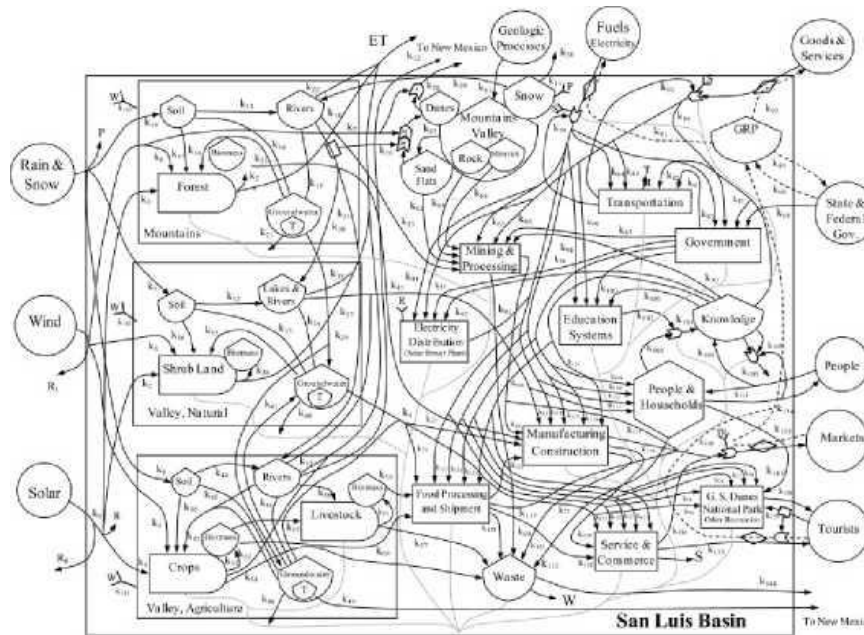


In this example, the 'additional construction' is that the vertices are marked with numbers and the edges have bolts on them.

Network theory is a tremendous, rambling subject. For example, it incorporates the investigation of electrical circuits:



In the 1950's, Howard Odum acquainted networks with model the progression of assets like energy through ecosystems:



Beginning, researcher have acquainted Systems bio image signs with depict systems. This is really 3 distinct dialects. For example, the Element Association Language allows you to discuss what substances mean for one another:

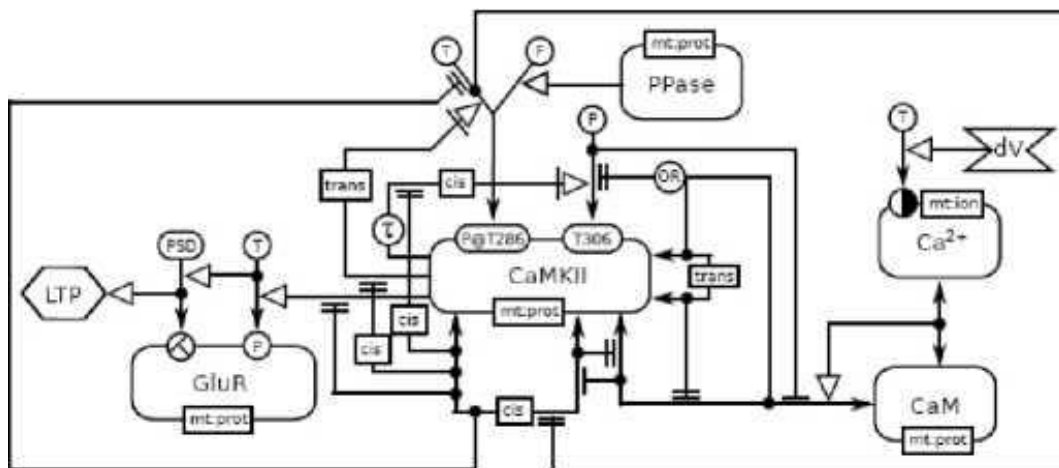
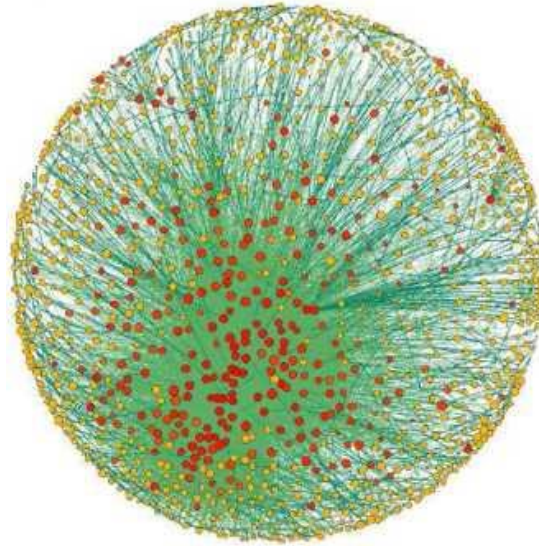


Figure 3: Guideline of calcium/calmoduhne kinase II impact on synaptic versatility.

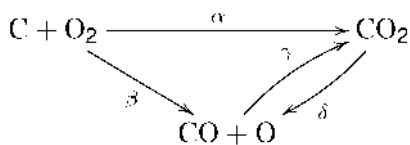
Many individuals use 'network hypothesis' to mean the investigation of enormous graphs, and how they exchange with time.



This is from a paper on "the network of worldwide corporate control", which examined possession joins among 600,000 organizations.

I have been dealing with 'response networks' and their applications to developing game theory — a point firmly associated with economics.

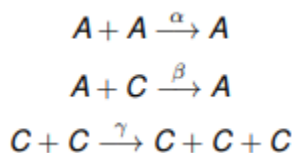
Response networks were brought into the world in science. Here is an example:



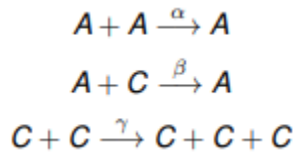
Here $\alpha, \beta, \gamma, \delta > 0$ are 'rate constants' for the Responses shown.

Response networks are likewise implied in developing game theory, a point significant in biology and economics.

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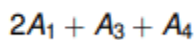
Response networks are likewise implied in developing game theory, a theme significant in biology and economics. For example, assume we have a populace of agents of two sorts: 'forceful' (A) and 'helpful' (C). Their dynamics may be portrayed by this Response network:



for certain constants $\alpha, \beta, \gamma > 0$. The thought is that forceful agents at times obliterate the agents they meet, while agreeable ones now and again replicate. We could expand this example endlessly by presenting more sorts of agents: for example, agents with various systems, areas, or assets.

All the more officially, to give a Response network we begin with any limited assortment of species A_1, A_2, \dots, A_k .

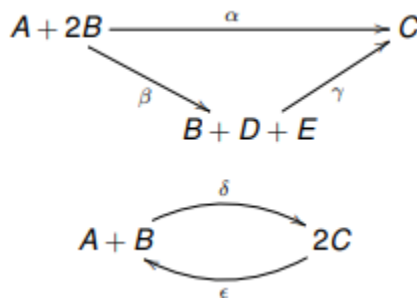
All the more officially, to give a Response network we start with any limited assortment of species A_1, A_2, \dots, A_k . We characterize a complex to be a straight blend of animal types with natural number coefficients, for example



We characterize a Response network to be a graph with:

- vertex marked by complexes
- Lines marked with bolts and furthermore certain rate given.

Here is an example of a Response network:



where $\alpha, \beta, \gamma, \delta, \epsilon$ are any sure numbers.

A Response network provide an developing game with stochastic dynamics. The thought is to record a vector ψ whose parts ψ_e are the possibility that the species adjacent are portrayed by some random complex l .

The thought is to record a vector ψ whose parts ψ_l are the probabilities that the species present are portrayed by some random complex l . Then, advance ψ as per the expert equation:

$$\frac{d\psi}{dt} = H\psi$$

The thought is to record a vector ψ whose parts ψ_{ℓ} are the probabilities that the species present are portrayed by some random complex ℓ . Then, advance ψ as per the expert equation:

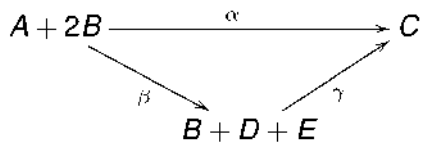
$$\frac{d\psi}{dt} = H\psi$$

Here H is a matrix whose sections depict the probabilistic rate at which one complex transforms into another. Along these lines, exhaustively:

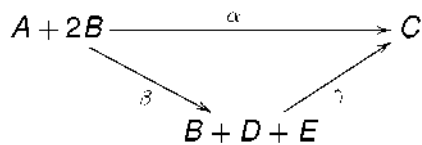
$$\frac{d\psi_{\ell}}{dt} = \sum_{\ell'} H_{\ell\ell'} \psi_{\ell'}$$

where $H_{\ell\ell'}$ is the probabilistic rate at which ℓ' becomes ℓ .

We can record the lattice sections $H_{\ell\ell'}$ beginning from the Response network by adhering to a few straightforward guidelines. For example, assume we have this Response network:

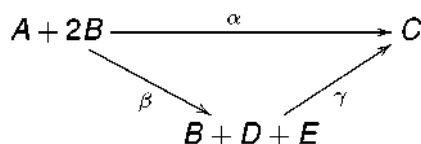


We can record the lattice sections $H_{\ell\ell'}$ beginning from the Response network by observing a few straightforward guidelines. For example, assume we have this Response network:



Suppose $\ell' = 5A + 3B + C$ and $\ell = 4A + B + 2C$.

We can record the lattice passages $H_{\ell\ell'}$ beginning from the Response network by adhering to a few straightforward guidelines. For example, assume we have this Response network:



Suppose $\ell' = 5A + 3B + C$ and $\ell = 4A + B + 2C$. Then

$$H_{\ell\ell'} = 5 \times 3 \times 2 \times \alpha$$

since the Response on top changes $A + 2B$ into C and there are 5 strategies for picking A and 3×2 structures for picking two B 's.

The experts have found wonderfully broad conditions under which the making game portrayed by a Response network has a remarkable concordance for each worth of the enormous number of made totals present. This outcome is known as the Need Zero Speculation.

The reasonable specialists Horne, Jackson and Feinberg have found wonderfully wide conditions under which the making game portrayed by a Response network has a glorious harmony for each worth of the enormous number of made totals present. This outcome is known as the Inadequacy Zero Speculation. The relentless nature of these equilibria is shown by finding a 'Lyapunov limit'. Overall, this induces showing that a specific full scale all that idea about decreases, and takes a base worth at the concordance.

In applications to science, this absolute is 'free energy'. Free energy everything considered diminishes, and takes its base worth together as one. This is a technique for managing saying that entropy pushes toward a greatest subject to impart targets.

In applications to science, this complete is 'free energy'. Free energy overall decreases, and recognizes its base worth as one. This is a method for managing saying that entropy pushes toward a greatest subject to unequivocal objectives. In unambiguous making games, this outcome is associated with Fisher's Essential Hypothesis on standard decision, which depicts how success increments through regular attestation.

In applications to science, this complete is 'free energy'. Free energy overall decays, and takes its base worth in balance. This is a strategy for regulating saying that entropy pushes toward a greatest subject to unequivocal necessities. In unambiguous making games, this outcome is associated with Fisher's Fundamental Speculation on common decision, which portrays how success increments through ordinary affirmation. In cash related applications, not genomes yet rather frameworks are being picked for.

A model: the 1-2-3 coordination game. In standard game speculation, this is a 2-player game where every player has 3 designs, and the two players win the accompanying settlements relying upon their decision of technique:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

An example: the 1-2-3 coordination game. In normal game theory, this is a 2-player game where every player has 3 systems, and the two players win the accompanying settlements relying upon their decision of technique:

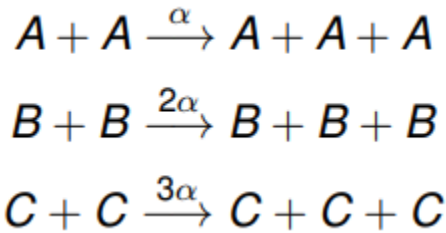
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

A model: the "1-2-3" coordination game. Similarly game speculation, this is a 2-player game where every player has 3 structures, and the two players win the going with settlements relying on their decision of reasoning:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

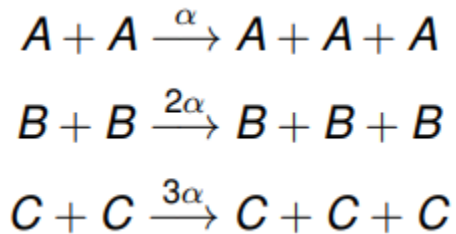
It's known as a coordination game since Nash equilibria with unadulterated procedures emerge when the two players pick a similar technique. There are likewise Nash equilibria with blended procedures.

Be that as it may, we should see this as an developing game given by this Response network:

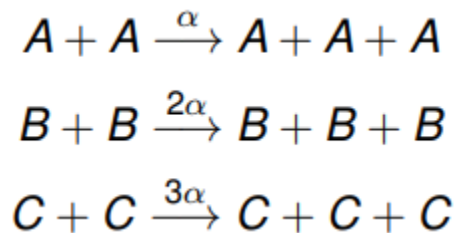


So, when players those approaches similar join every furthor, they can increase.

But let's view this as an emerging game specified by this Response network:



However, how about we notice this as an developing game given by this Response network:



CONCLUSION

Mathematical modeling has turned into a foundation in the analysis of complex systems, offering priceless bits of knowledge into the way of behaving and dynamics of systems with various communicating parts. All through this paper, we have investigated different mathematical methodologies, including agent-based models (ABMs), and network theory, each contributing exceptionally to how we might interpret complexity.

The thought about innovation of mathematical modeling is presented as entire instruments, methods, information for mathematical model piece, ID, check, and double-dealing. The thoughts about 'basic' and 'complex' systems (processes, peculiarities) are presented as associated firmly with mathematical modeling innovation advancement. Reproduction models are treated as being on the boondocks among 'straightforward' and 'complex' peculiarities (processes systems). This wilderness is thought of as moving from 'easy' to 'complex' or from 'mathematical' to 'humanities'. Different parts of association among mathematical and humanities methods of forecast are portrayed.

REFERENCES

1. N. V. Martyushev, B. V. Malozyomov, S. N. Sorokova, E. A. Efremkov, D. V. Valuev, and M. Qi, "Review models and methods for determining and predicting the reliability of technical systems and transport," *Mathematics*, vol. 11, no. 15, p. 3317, 2023.
2. A.G. Gad, "Particle swarm optimization algorithm and its applications: a systematic review," *Archives of computational methods in engineering*, vol. 29, no. 5, pp. 2531-2561, 2022.
3. H. Sarker, "Ai-based modeling: Techniques, applications and research issues towards automation, intelligent and smart systems," *SN Computer Science*, vol. 3, no. 2, p. 158, 2022.
4. N. Sharma and P. Gardoni, "Mathematical modeling of interdependent infrastructure: An object-oriented approach for generalized network-system analysis," *Reliability engineering & system safety*, vol. 217, p. 108042, 2022.
5. Q. Zhang and Y. Zhou, "Recent advances in nonGaussian stochastic systems control theory and its applications," *International Journal of Network Dynamics and Intelligence*, pp. 111-119, 2022.
6. S. A. Nugroho, A. F. Taha, N. Gatsis, and J. Zhao, "Observers for differential algebraic equation models of power networks: Jointly estimating dynamic and algebraic states," *IEEE transactions on control of network systems*, vol. 9, no. 3, pp. 1531-1543, 2022.
7. A. El-Awady and K. Ponnambalam, "Integration of simulation and Markov Chains to support Bayesian Networks for probabilistic failure analysis of complex systems," *Reliability Engineering & System Safety*, vol. 211, p. 107511, 2021.
8. D. Baleanu, S. S. Sajjadi, A. Jajarmi, and Ö. Defterli, "On a nonlinear dynamical system with both chaotic and nonchaotic behaviors: a new fractional analysis and control," *Advances in Difference Equations*, vol. 2021, no. 1, pp. 1-17, 2021.
9. I.C. Clarke, "Cellular automata and agent-based models," in *Handbook of regional science*: Springer, 2021, pp. 1751-1766.
10. I.Schoenenberger, A. Schmid, R. Tanase, M. Beck, and M. Schwaninger, "Structural analysis of system dynamics models," *Simulation Modelling Practice and Theory*, vol. 110, p. 102333, 2021.
11. R. I. Sujith and V. R. Unni, "Dynamical systems and complex systems theory to study unsteady combustion," *Proceedings of the Combustion Institute*, vol. 38, no. 3, pp. 3445-3462, 2021.
12. W. Fan, P. Chen, D. Shi, X. Guo, and L. Kou, "Multiagent modeling and simulation in the AI age," *Tsinghua Science and Technology*, vol. 26, no. 5, pp. 608-624, 2021.
13. A.F. Siegenfeld and Y. Bar-Yam, "An introduction to complex systems science and its applications," *Complexity*, vol. 2020, pp. 1-16, 2020.
14. A.S. Currie et al., "How simulation modelling can help reduce the impact of COVID-19," *Journal of Simulation*, vol. 14, no. 2, pp. 83-97, 2020.
15. S. R. Nilsson et al., "Simple Behavioral Analysis (SimBA)—an open source toolkit for computer classification of complex social behaviors in experimental animals," *BioRxiv*, p. 2020.04. 19.049452, 2020.
16. R. Sameni, "Mathematical modeling of epidemic diseases; a case study of the COVID-19 coronavirus," *arXiv preprint arXiv:2003.11371*, 2020.
17. Ernesto Estrada, *The Structure of Complex Networks: Theory and Applications*, Oxford U. Press, 2019.
18. John Baez, Jacob Biamonte and Brendan Fong, *Network Theory*, 2019.

19. William Sandholm, Developing Game Theory, 2017.
20. Jonathan Guberman, Mass Action Response Networks and the Deficiency Zero Theorem, B.A. thesis, Department of Mathematics, Harvard University, 2016.