Algebraic Properties of $\boldsymbol{PGL_2}(\mathbb{C})$ for Long Exact Fibration Sequence with Sporadic Extensions[1]

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DOI:10.37648/ijrst.v13i02.001

Received: 19 February 2023; Accepted: 18 April 2023; Published: 19 April 2023

ABSTRACT

A concise formulation is given regarding the constructions of the group $PGL_2(\mathbb{C})$ with its related algebraic properties with intertwined topological aspects in the long exact fibration sequences as considered over homotopy and higher order homotopy groups with further extension to sporadic groups including the monster group formulations.

Keywords – *Lie Groups; Homotopy.*

Construction of $\boldsymbol{PSL_2}(\mathbb{C})$

Taking an affine Lie algebra $\mathcal{L}$ for the expressed automorphisms conjugate over a parameterization ζ_j over such that for $(1 \leq j \leq i)$ such that there exists the $Projective\ General\ Linear\ Group\ PGL$ where $\mathcal{L}$ acting on $\mathbb{C}$ for the characterization of $\bar{\bar{\bar{\zeta}}}_{i-1} \exists i \geq 2$ relates $PGL_{\bar{\bar{\zeta}}}(\mathbb{C})$ for $\bar{\bar{\bar{\zeta}}}_k^{(1)}\ \exists k = i - 1 + 1\ \forall i = 2$ in $PGL_k(\mathbb{C})$ we get the relation for $\mathcal{L}$ on $\mathbb{C}$ for $k = 2$ and the generator $\mathcal{G}$ in $(1 \leq j \leq i)$ for $i \neq j$[1,8].

Taking a vector space v for the generator $\mathcal{G}$ which generates the $PGL(v)\ \exists PGL$ arises for all the non-zero scaler transformations Z of the subgroup $\mathfrak{g}$ of $\mathcal{G}$ such that for v there exists[2,8,9],

1. Scalar transformations $Z(v)$
2. General linear group $GL(v)$
3. Projective general linear group $PGL(v)$
    a. For a relation $PGL(v) = GL(v)/\ Z(v)$
    b. Where for the projective special linear group $PSL(v)$

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[1] *How to cite the article:* Bhattacharjee D., Apr-Jun 2023; Algebraic Properties of $\boldsymbol{PGL_2}(\mathbb{C})$ for Long Exact Fibration Sequence with Sporadic Extensions; *International Journal of Research in Science and Technology*, Vol 13, Issue 2, 1-9, DOI: http://doi.org/10.37648/ijrst.v13i02.001

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i. The explicit relation exists in a same way as

$$PSL(v) = \frac{SL(v)}{SZ(v)} \exists SZ(v) \subset^{Group} Z(v) \subset^{Group} \mathcal{G}$$

$$\xrightarrow{\cong} SZ(v) \subset^{Group} \mathfrak{g} \subset^{Group} \mathcal{G}$$

Where one gets the automorphism of $\mathcal{G}$ represented as $Aut(G) \simeq Aut_{\mathbb{C}}$ for the algebraic structure $PGL_{j+1}(\mathbb{C}[\gamma^n]) \exists \gamma^n \ni \gamma^1, \gamma^{-1}$ has the structure for $k = 2$[3,9],

$$Aut_{\mathbb{C}}(\mathbb{C}[\gamma^n]) \ltimes PGL_k(\mathbb{C}[\gamma^n])$$

For a mapping,

$$\begin{array}{ccccccc}
SZ(v) & \hookrightarrow & Z(v) & & & & \\
\downarrow & & \downarrow & & & & \\
SL(v) & \hookrightarrow & GL(v) & & & & \\
\downarrow & & \downarrow & & & & \\
PSL(v) & \hookrightarrow & GL(v) & \longrightarrow & v & \xrightarrow{generates\ the\ generator\ of\ group} & \mathcal{G}
\end{array}$$

For $G \subset PGL_2(\mathbb{C})$ such that for $GL_2(\mathbb{C})$and $SL_2(\mathbb{C}) -$ there exists a computable relation for $ad - bc = 1$ in $GL_2(\mathbb{C})$ such that,

$$PGL_2(\mathbb{C}) \xrightarrow{isomorphic\ to} PSL_2(\mathbb{C}) \exists$$

$$SL_2(\mathbb{C})/\{\pm I\} = PGL_2(\mathbb{C}) \Longrightarrow ad - bc = 1$$

$$\forall GL_2(\mathbb{C}) \ni \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Where,

$$GL_2(\mathbb{C})/\mathbb{C} * I = PGL_2(\mathbb{C}) \exists GL_2(\mathbb{C}) \xrightarrow{\Rightarrow} z \longmapsto \frac{az + b}{cz + d}$$

$PSL_2(\mathbb{C})$ as in algebraic topology

Considering a topological space S of all continuous functions, a mapping parameter ι is defined for the fibre $f_\circ$ such that the total space $f_\circ$ acts on the base $\widetilde{f_\circ}$ over a representation σ for the union of group homeomorphisms in the form,

$$\mathcal{G}_m \xrightarrow{\sqcup_1} \mathcal{G}_{m+1} \xrightarrow{\sqcup_2} \mathcal{G}_{m+2}, \dots \dots, \xrightarrow{\sqcup_n} \mathcal{G}_{m+n}$$

For the $\mathcal{G}_l \exists l \ni m, m+1, m+2, \dots\dots, m+n$ where the image of each homeomorphism $\sqcup_*$ equates the kernel of the next homeomorphism represented by,

$$\ell_{ex} = (\sqcup_* \equiv Ker(\sqcup_{\bar{\bar{*}}}) \ni \bar{\bar{*}} = * + 1)$$

Where for the three parameters,

1. The mapping parameter ι
2. The topological space as considered earlier S
3. With its pair space $\bar{S}$
   a. For $\bar{S} \subseteq S$
      i. In the map $\sigma : f_\circ \longrightarrow \widetilde{f_\circ}$
      ii. For the representation σ,

$$\bigcup_{\substack{h_E \ni (S,\bar{S},\sigma) \\ \ell_{ex} \xrightarrow{acts\ on}}} \left(h_E \bigcup \ell_{ex}\right) \Bigg| \text{for extension}$$

Gives the homotopy lifting for,

$$\sigma : f_\circ \longrightarrow \widetilde{f_\circ} \ \exists \sigma \text{ for Serre fibration}$$

Over a **k** – skeleton in S representing,

$$\{S_{\boldsymbol{k}}\}$$

For a 'closure–finite weak topology' in the criterion,

Through a gluing parameter $\mathcal{I}$ gluing k – cells for S_k for a union of a sequence of S_k,

$$\exists \text{ this denotes a union to obtain } S_k \text{ from } S_{k-1}$$

For each cell C to glue upto k – cells C_k via gluing parameter $\mathcal{I}$ for the topological space $S_{k-1} to\ S_k$ through the union of long exact sequence ℓ_{ex} represented via[3-6,9],

$$\bigcup \ell_{ex} \left(\bigcup_{\{\mathcal{I}\} \, {}^{S_k}_{S_{k-1}} \text{for} \, \cup C_k} C_k^{\mathcal{I}} \right) \equiv \Delta_{CW}$$

Restricting to a particular variety of $PSL_2(\mathbb{C})$ that is for the natural number $\epsilon \geq 2$ in $\mathbb{P}S^{\epsilon}(\mathbb{C}^2)$ it is easy to define $S^{\epsilon}(\mathbb{C}^2) \cong S^2(\mathbb{C}^2)^{\vee}$ for $\epsilon = 2$ a decomposition can be made into submodules,

1. For the parameter representing decomposition,
$$S^2 S^{\epsilon}(\mathbb{C}^2)$$
2. The submodule representation,

$$S^2 S^{\epsilon}(\mathbb{C}^2) = \bigotimes_{u \geq 0} S^{2r-4u}(\mathbb{C})^2$$

Thus the homotopy lifting property can be satisfied for homotopy $\tilde{\sqcup}$: $\tilde{S} \times [0,1] \rightarrow \widetilde{f_\circ}$ and $\sqcup$: $S \times [0,1] \rightarrow f_\circ$ for a covering of $\neg$ extending $\bar{\bar{\sqcup}}$ $\exists for\ (S, \bar{S}, \sigma)$ of the lifting extension there is the map $\sigma : f_\circ \longrightarrow \widetilde{f_\circ}$ $\exists \sigma$ for Serre fibration with the CW – complex relations Δ_{CW} when $\sqcup |_{\neg} \equiv \bar{\bar{\sqcup}}$ takes place one can get the homotopy lifting for $\{S\}, \sigma$ for $\sqcap \equiv S \times \{0\}$ for $\tilde{S} = \Phi$ into,

$$\neg := \tilde{\sqcup} \subseteq \sqcup \bigcup \sqcap$$

For the mapping that's taken for the Serre fibration as,

$$\sigma : f_\circ \longrightarrow \widetilde{f_\circ} \; \exists \sigma \text{ for Serre fibration}$$

If $\widetilde{f_\circ}$ which is the base can be replaced by a parameter $\overline{\overline{f_\circ}}$ such that $\overline{\overline{f_\circ}}$ is a topological space such that for $f \in \overline{\overline{f_\circ}}$ there exists a open neighbourhood $\mathcal{O} \subset \overline{\overline{f_\circ}}$; the covering of $\overline{\overline{f_\circ}}$ is a continuous map only for the spaces having isolated points as such,

- For the space $\overline{\overline{f_\circ}}$ having a metric F one can define such discrete spaces ρ via points Λ_0 and Λ_1 where $\left(\overline{\overline{f_\circ}}, F\right)$ is a discrete space of isolated points of the metric Λ_0, $\Lambda_1 \in F$ such that,
  - For 1 $\longrightarrow \Lambda_0 \neq \Lambda_1$
  - For 0 $\longrightarrow \Lambda_0 = \Lambda_1$
    - Leading to a map of covering space,
      $\overline{\overline{\sigma}} : f_\circ \longrightarrow \overline{\overline{f_\circ}}$

      $$\xrightarrow{\text{for a disjoint union}} \frac{1}{\overline{\overline{\sigma}}}(\mathcal{O}) = \coprod_{P \in \rho} V_P$$
    - For a homeomorphism relating $\mathcal{O}$ to every element P of ρ.

This with considering the higher order homotopy groups for the topological space M as the n^{th} order as $\pi_n(M)$ shows the structured map,

$$\begin{array}{ccc} SL & \xrightarrow{\quad 2n,\, 1+\mathbb{R} \quad} & PSL \\ \big\downarrow & & \big\downarrow \\ GL & \xrightarrow{\quad n,\, \mathbb{C} \quad} & PGL \end{array}$$

Sporadic Groups

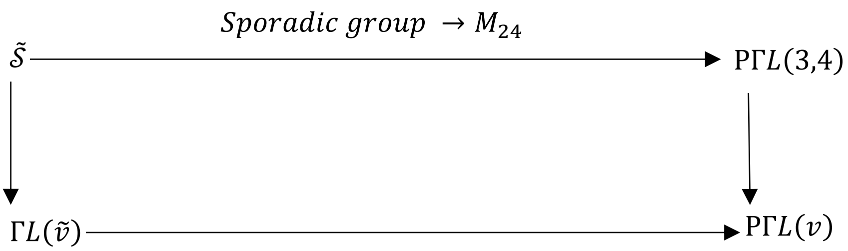
While isomorphism preserves the structure by inverse mapping, the automorphism is a special kind of isomorphism which maps onto itself by means of a symmetry preserving grouping structure called the automorphism group. Considering the automorphism; when one takes a field parameterized by δ with two vectors v_1 and v_2 acting on the field δ such that for an image ∂_* – a semilinear map can be defined for $*$ acting on the scaler ∂ over a for the vector summation v_n defined[3-7],

$$\sum_{n=1}^{2} v_n$$

There exists a semilinear group $\Gamma L(\tilde{v})$ for the field –automorphism of δ or in the special case of complex $\delta_{\mathbb{C}}$ when the $* -$ semilinear goes for a semilinear transformations $\mathcal{S} : \tilde{v} \longrightarrow \tilde{v}$ then for non–zero $\mathcal{S}$ one gets the representation of the semilinear groups for,

$$\mathcal{S}\partial_{*(\delta,\delta_{\mathbb{C}})} \exists \mathcal{S} \neq 0 \xrightarrow{\quad \Rightarrow \quad} \Gamma L(\tilde{v})$$

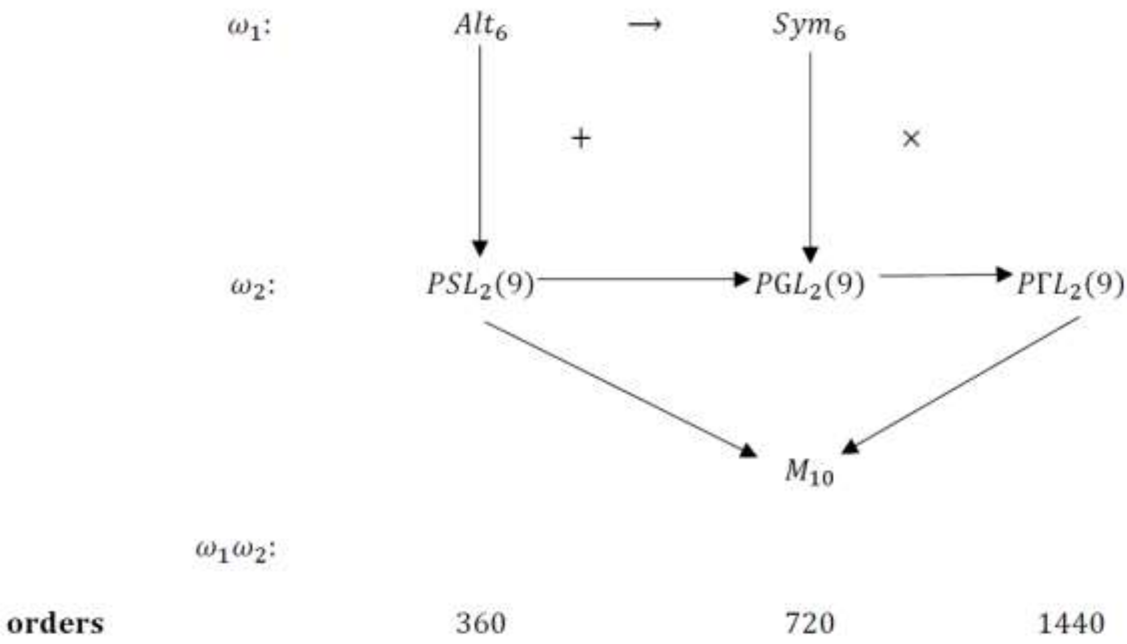
Thus, for $\tilde{S} := \tilde{v} \longrightarrow \bar{v}$ for a denoted projection of $\tilde{v}$ to $\bar{v}$: it is easy to get a projective semilinear group from $\tilde{S} := PG(\tilde{v}) \longrightarrow PG(\bar{v})$ such that one can define the action η for higher order groups like the projective semilinear groups from semilinear groups through the blow mapping,



Sporadic groups come in 3 $families\ +1$ that is under 4 $classes$ which is given as[7,10,11],

$classes$		
	$Mathieu$	$5\ groups$
	$Leech$	$7\ groups$
	$Monster$	$8\ groups$
	$Pariah$	$6\ groups$

The Mathieu groups are as $- M_{11}, M_{12}, M_{22}, M_{23}, M_{24}$. From [ ] one gets a way of the construction of $M_{11}\ and\ M_{12}$ where the $2-groups\ Alt_6$ and $PSL_2(9)$ are considered with an isomorphism between them for the natural 2-extension of $Alt \cdot 2 = Sym_n$ with $2-natural$ one $PSL_2(m)$ and $PGL_2(m)$ with $2-automorphism$ class ω_1 and $\omega_2 -$ involutory and commute making together $(\omega_1, \omega_2, \omega_1\omega_2)$ giving the below chart with $P\Gamma L_2(9)$ refers to an involuntary automorphism of order $1440 - v_9 \exists for\ m = 3, v = 2, m^v = 3^2 = 9$ where $+$ beside the arrow represents $isomorphism$ and $\times$ beside the arrow represents $no\ isomorphism$,

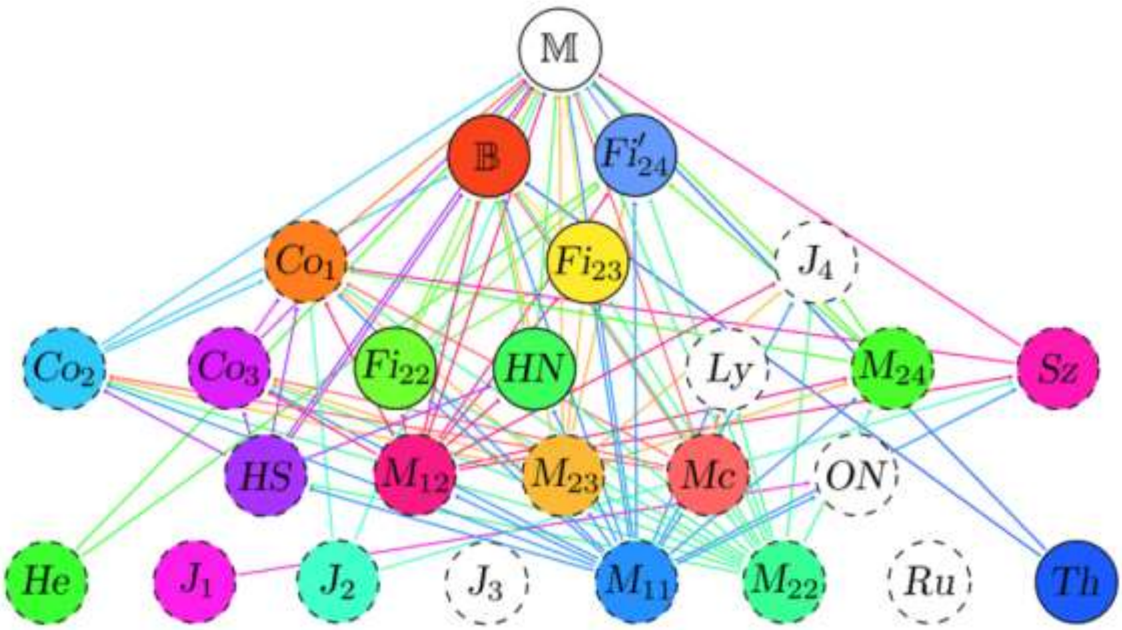


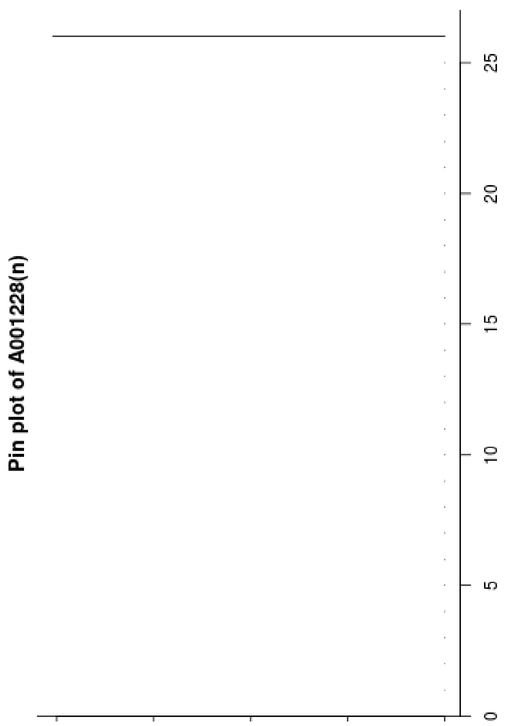
The other $3 - groups$ are[8-11,12],

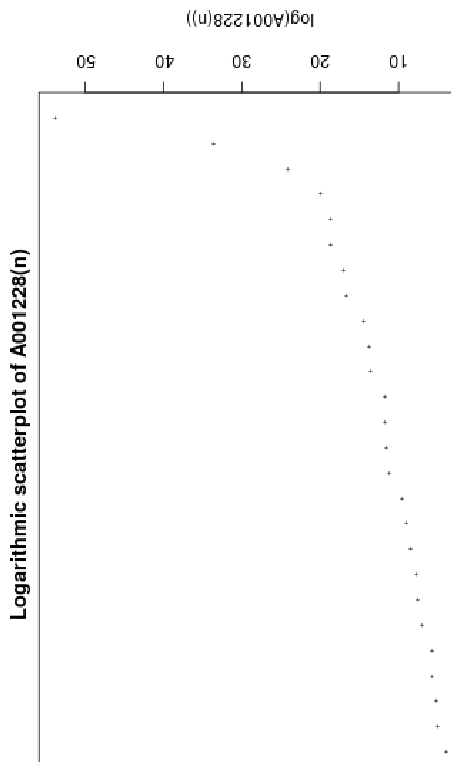
$groups$		
	$Leech$	Co_1, Co_2, Co_3, Suz, McL, HS, J_2
	$monster$	M, F_2, F_3, F_5, F_7, Fi_{24}, Fi_{23}, Fi_{22}
	$pariahs$	J_1, J_3, J_4, $O'N$, Ru, Ly

(IJRST) 2023, Vol. No. 13, Issue No. 2, Apr-Jun e-ISSN: 2249-0604, p-ISSN: 2454-180X

Order of Sporadic Groups Depiction as – Plot: Graph: Table[11-14]







A001228 as a simple table

n	a(n)
1	7920
2	95040
3	175560
4	443520
5	604800
6	10200960
7	44352000
8	50232960
9	244823040
10	898128000
11	4030387200
12	145926144000
13	448345497600
14	460815505920
15	495766656000
16	42305421312000
17	64561751654400
18	273030912000000
19	51765179004000000
20	90745943887872000
21	4089470473293004800
22	4157776806543360000
23	86775571046077562880
24	1255205709190661721292800
25	4154781481226426191177580544000000
26	808017424794512875886459904961710757005754368000000000

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[7920,95040,175560,443520,604800,10200960,
 44352000,50232960,244823040,898128000,4030387200,
 145926144000,448345497600,460815505920,
 495766656000,42305421312000,64561751654400,
 273030912000000,51765179004000000,
 90745943887872000,4089470473293004800,
 4157776806543360000,86775571046077562880,
 1255205709190661721292800,
 4154781481226426191177580544000000,
 80801742479451287588645990496171075700575436800000\
 0000]
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