WAVE PARTICLE DUALITY IN NEWTONIAN MECHANICS

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ABSTRACT

Wave particle duality is one of the most fundamental ideas that involves in the development of quantum mechanics. Louis de Broglie derived wave particle duality relation using spacial relativity theory and wave mechanics. In this article wave nature of a moving particle is investigated and explained using Newtonian mechanics only. It is observed that in this universe even a fully symmetric and spin free particle may move in a linear motion only in a force-field free region. Using Newtonian mechanics a relation between wave length and momentum of a moving particle is obtained which is equivalent to de Broglie's relation. But, proportionality constant, b, used in present relation, depends on force-field and spin moment of the moving particle. Wave length of a moving particle according to the present theory may be equivalent to de Broglie's wave length or may be a correction term to de Broglie's wave length due to field perturbation, is a matter of debate.

Keywords: *Wave particle duality; Newtonian mechanics; symmetric particle; linear motion; force-field*

INTRODUCTION

Wave particle duality (de Broglie, 1925) is one of the fundamental laws that involves in the development of quantum mechanics in the early stage of twentieth century. Louis de Broglie in his Ph. D. thesis work use Einstein's theory of special relativity (Einstein, 1917) and equation of wave motion to find wave nature of a moving particle. L. de Broglie made a "**very natural assumption**" (Weinberger, 2006) that if w be the velocity of a "**light quantum**" of frequency v, where w is "**nearly close to Einstein's limiting velocity**" c, and let us assume that all such "**light quanta**" are of the same mass m_0. Then the energy W of one such quantum has to fulfill the relation

$$W = hv = m_0 c^2/(1-\beta^2)^{1/2}, \beta = w/c \quad (1)$$

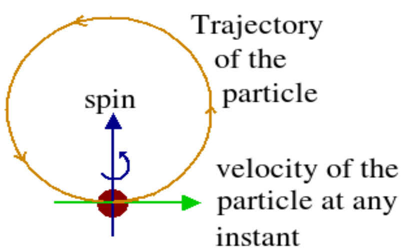
Finally de Broglie arrived at the famous equation which relates momentum(p) and wave length(λ) of a moving particle as

$$p = h / \lambda \quad (2)$$

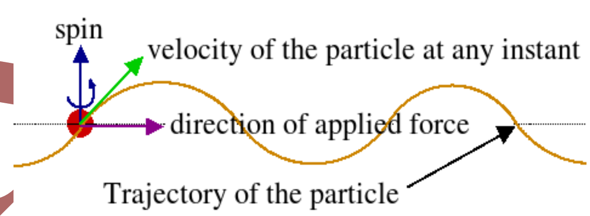
where h is plank's constant. Till now, nature of wave associated with a moving particle according to equation 2 is not explained, i.e. whether it is matter wave or light wave. In this article Newtonian mechanics is used to study the nature of the trajectory of a particle moving through a force-field. With out application of any quantization condition or spacial relativity approximation we may reach to a similar equation of momentum for a moving particle which suggests that the wave associated with a moving particle is a trajectory i.e. a matter wave and should have wave velocity less than the velocity of light.

NATURE OF TRAJECTORY OF A MOVING PARTICLE IN ANY FORCE-FIELD

When any particle moves through any force-field it should perturbs the field or in the other sense we can say that no particle can move with out perturbing a field. This phenomenon is as simple as a particle moves through any medium. When a particle moves through a medium, particles of the medium must move accordingly. If a particle have only spin, neither the particle nor the medium has translational motion, trajectory of the particle should be circler as shown in Figure-(a). Radius is inversely proportional to the momentum of the particle. But, if there is a constant force acting on the particle such that the particle moves with a constant velocity *u* which is perpendicular to the direction of spin, the trajectory of the particle would be as *sin θ* as shown in Figure-(b). When a particle moves through a force-field similar situation arises.



(a) free spning particle



(b) forced spning particle

Let a particle of mass *m* is moving with a velocity *u* in a force-field *F*. Let F_1 is opposition force and θ is the angle between force-field (F) and opposition field (F_1). Then, resultant force F_r would be as follows -

$$F_r = F \times F_1 \tag{3}$$

or

$$F_r = |F| \cdot |F_1| \sin \theta \tag{4}$$

or

$$F_r = A \sin \theta \tag{5}$$

where, $A = |F| \cdot |F_1|$, amplitude of the sine wave; since $|F| \cdot |F_1|$ is a number, not a vector quantity. Thus, if $\theta \neq 0$, the trajectory of the particle would be a sine wave. In general, for opposition force like frictional force, $\theta = 0$, but for any asymmetric particle or a particle with spin $\theta \neq 0$. Hence, we can say- ***motion of any asymmetric particle or a particle with spin should have wave nature even the applied force-field is linear***. If this is the origin of wave nature of a moving particle then any spin-less fully symmetric particle should not show any wave nature in any linear force-field. In de Broglie's wave particle duality, wave nature of a moving particle is not governed by the spin or symmetry of the particle. That is the window through which present theory (or one can say a hypothesis) may be separated out from de Broglie's theory or existence of any wave nature of a moving particle as it is considered in this work, may be justified.

WAVE LENGTH OF A MOVING PARTICLE

In previous section we have found that motion of any asymmetric or spinning particle should be wave like, we may say this wave as **matter wave**. Let us now find the relation between the wave length (λ) and velocity (u) of a moving particle.

From Equation -5 we get the resultant force which is acting on the particle. Thus, if m be the mass of the particle, then acceleration (f) of the particle would be $(A/m)\sin\theta$. As the particle moves in a wave like path, its velocity (u) along the direction of its propagation would change with time which is nothing but the acceleration of that particle. Thus, we can write -

$$f = du/dt = (A/m)\sin\theta \quad (6)$$

and,

$$du = (A/m)\sin\theta\, dt \quad (7)$$

From Figure-b, it is obvious that θ is also function of time (t). Let find the values of u and θ at different time interval. At $t = 0; u = 0$ and $\theta = \theta_i$; θ_i is the initial angle between force-field (or direction of propagation) and resultant force. At $t = T/4$ where T is the time period of the wave ; $u = u_{max}$ and $\theta = 0$. Thus, on integration of Equation-7 from $t = 0$ to $T/4$ we get -

$$u_{max} = \int_0^{T/4} (A/m)\sin\theta\, dt \quad (8)$$

In Equation-8 θ is a function of t which is as follows -

$$\theta = \tan^{-1}(\cos t) \quad (9)$$

u_{max} is the velocity of the wave, u. From Equation-9 we get -

$$dt = -(1/\sqrt{(1-\tan^2\theta)}\sec^2\theta\, d\theta \quad (10)$$

Putting the value of dt in Equation-8, we get -

$$u = -\int_{\pi/4}^{0} (A/m)\sin\theta(1/\sqrt{(1-\tan^2\theta)}\sec^2\theta\, d\theta \quad (11)$$

On integration we get

$$mu = A\pi/4 \quad (12)$$

mu is the momentum of the particle, p. If T be the time period of the wave, $\pi = T/2$ where $T = 1/\lambda$, λ is the wave length of the wave we are dealing here. Using above mentioned relations we get

$$p = b/\lambda \quad (13)$$

where, $b = A/8$. Equation-2 and Equation-8 are equivalent except h is replaced by b. h is Plank's constant which is 6.626 $X10^{-27}$ erg-sec. b depends on amplitude of the wave, A, which depends on both applied force-field and spin moment (or degree of asymmetry). In this respect h and b are different.

COMPARISON OF WAVE PARTICLE DUALITY IN DE BROGLIE'S METHOD AND PRESENT METHOD

In de Broglie's duality it is stated that every moving particle has a wave nature but nature of wave is not mentioned. In present work, it has been shown that the wave related to a moving particle is the trajectory of the particle. In de Broglie's relation, wave length (λ) is independent of the force-field where it moves. But according to the present theory wave length, λ, of a particle should depends on the force-field and spin moment. A spin less and symmetric particle may have de Broglie wave length but according to the present method its matter wave should be 0. The value of constant b is not determined. Experimental value of b would be helpful to correlate present theory with de Broglie's theory.

CONCLUSION

In this article wave nature of a moving particle is explained using Newtonian mechanics. With out using spacial relativity de Broglie type relation is obtained. It is found that wave length of a moving particle is inversely proportional to its momentum which is equivalent to de Broglie's wave particle duality relation. But present method is different from de Broglie's theory with respect to the constant term. In de Broglie's relation constant term is h, Plank's constant. But, in the present method constant b is not a universal constant. Its value depends on the spin momentum and force-field. de Broglie's theory is silent about the spin of the particle. But according to the present theory every moving particle should have spin. At present it is not clear whether this wave length is equivalent to de Broglie's wave length or it is a correction term to de Broglie's wave length due to the presence of force field. An experimental verification would justify the truth. If experiment suggests that both are equivalent then we can say de Broglie's wave nature of a particle is a matter wave. It also suggests that wave nature and particle nature may exist at a same time. If experiment suggests that these two theories are not equivalent and value of the de Broglie's wave length of any moving particle is larger than that calculated using present theory, we may say that wave length calculated using this theory is a correction term to de Broglie's wave length due to the force-field perturbation. At that situation it may be considered that de Broglie wave length of a moving particle is not a trajectory. It may be something else.

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