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LIAPUNOV'S STABILITY THEORY-BASED MODEL REFERENCE ADAPTIVE CONTROL FOR DC MOTOR

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ABSTRACT

Fractional order dynamic model could model various real materials more adequately than integer order ones and provide a more adequate description of many actual dynamical processes. Fractional order controller is naturally suitable for these fractional order models. In this paper, a fractional order PID controller design method is proposed for a class of fractional order system models. Better performance using fractional order PID controllers can be achieved and is demonstrated through two examples with a comparison to the classical integer order PID controllers for controlling fractional order systems.

Index Terms—Fractional order calculus, fractional order controller, fractional order systems, PIλDμ controller.

INTRODUCTION

The concept of extending classical integer order calculus to non-integer order cases is by no means new. For example, it was mentioned in [1] that the earliest systematic studies seem to have been made in the beginning and middle of the 19th century by Liouville, Riemann, and Holmgren. The most common applications of fractional order differentiation can be found in [2]. The concept has attracted the attention of researchers in applied sciences as well. There has been a surge of interest in the possible engineering application of fractional order differentiation. Examples may be found in [3] and [4]. Some applications including automatic control are surveyed in [5].

In the field of system identification, studies on real systems have revealed inherent fractional order dynamic behavior. The significance of fractional order control is that it is a generalization of classical integral order control theory, which could lead to more adequate modeling and more robust control performance. Reference [6] put forward simple tuning formulas for the design of PID controllers. Some MATLAB tools of the fractional order dynamic system modelling, control and filtering can be found in [13]. Reference [7] gives a fractional order PID controller by minimizing the integral of the error squares. Some numerical examples of the fracti controller was designed to ensure that the closed-loop system is robust to gain variations and the step

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responses exhibit an iso-damping property. For speed control of two-inertia systems, some experimental results were presented in [12]by using a fractional order PI$^\alpha$D controller. A comparative introduction of four fractional order controllers can be found in [10].

In most cases, however, researchers consider the fractional order controller applied to the integer order plant to enhance the system control performance. Fractional order systems could model various real materials more adequately than integer order ones and thus provide an excellent modeling tool in describing many actual dynamical processes. It is intuitively true, as also argued in [11], that these fractional order models require the corresponding fractional order controllers to achieve excellent performance. In this paper, a fractional order PID controller is used to control a class of fractional order systems. A fractional order PID controller design method is proposed with two illustrative examples.

The remaining part of this paper is organized as follows: in Sec. II, mathematical foundation of fractional order controller is briefly introduced; in Sec. III, the fractional order PID controller and its property are presented; in Sec. IV, the fractional order PID controller parameter setting is proposed with specified gain and phase margins; in Sec. V, two examples are presented to illustrate the superior performance achieved by using fractional order controllers. Finally, conclusions are drawn in Sec. VI.onal orders were presented in [8]. In [9], a PI$^\alpha$

II. A BRIEF INTRODUCTION TO FRACTIONAL ORDER CALCULUS

A commonly used definition of the fractional differ-integralis the Riemann-Liouville definition

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)}\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)^m \int_a^t \frac{f(\tau)}{(t-\tau)^{1-(m-\alpha)}}\mathrm{d}\tau \quad (1) $$

for $m - 1 < \alpha < m$ where $\Gamma(\cdot)$ is the well-known Euler's gamma function. An alternative definition, based on the concept of fractional differentiation, is the Gr¨unwald- Letnikov definition given by

$$ {}_aD_t^\alpha f(t) = \lim_{h\to 0}\frac{1}{\Gamma(\alpha)h^\alpha}\sum_{k=0}^{(t-a)/h}\frac{\Gamma(\alpha+k)}{\Gamma(k+1)}f(t-kh). \quad (2) $$

One can observe that by introducing the notion of fractional order operator ${}^{aD\alpha}{}_t$ $f(t)$, the differentiator and integrator can be unified. Another useful tool is the Laplace transform. It is shown in [14] that the Laplace transform of an n-th derivative ($n \in R+$) of a signal $x(t)$ relaxed at $t = 0$ is given by:L_$Dnx(t)$ _= $snX(s)$. So, a fractional order differential equation, provided both the signals $u(t)$ and $y(t)$ are relaxedat $t = 0$, can be expressed in a transfer function form $(t - \tau)1-(m-\alpha)$ dτ (1)A simple model of a DC motor driving an inertial load shows the angular rate of the load,$\omega(t)$, as the output and applied voltage, $v_{app}(t)$, as the input. This picture shows a simple model of the DC motor. In this model, the dynamics of the motor itself are idealized; for instance, the magnetic field is assumed to be constant. The resistance of the circuit is denoted by R and the self-inductance of the armature by L. The important thing here is that with this simple model and basic laws of physics, it is possible to

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develop differential equations that describe the behavior of this electromechanical system. In this, the relationships between electric potential and mechanical force are Faraday's law of induction and Ampere's law for the force on a conductor moving through a magnetic field.

2.1) Mathematical Derivation: The torque τ, seen at the shaft of the motor is proportional to the current i induced by the applied voltage,

$$\tau(t) = K_m i(t) \dots\dots(29)$$

Where K_m, the armature constant, is related to physical properties of the motor, such as magnetic field strength, the number of turns of wire around the conductor coil, and so on.

The back (induced) electromotive force, v_{emf}, is a voltage proportional to the angular rate ω seen at the shaft,

$$v_{emf}(t) = K_b \omega(t) \dots\dots(30)$$

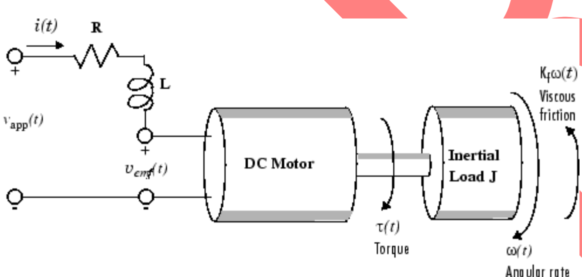
Where K_b, the emf constant, also depends on certain physical properties of the motor.

The mechanical part of the motor equations is derived using Newton's law, which states that the inertial load J times the derivative of angular rate equals the sum of all the torques about the motor shaft. The result is this equation,

$$J\frac{d\omega}{dt} = -K_f \omega(t) + K_m i(t) \dots\dots(31)$$

Where $K_f \omega$ is a linear approximation for viscous friction.

Finally, the electrical part of the motor equations can be described by



Finally, the electrical part of the motor equations can be described by

$$v_{app}(t) - v_{emf}(t) = L\frac{di}{dt} + Ri(t) \dots\dots(32)$$

Substituting for the back emf

$$v_{app}(t) = L\frac{di}{dt} + Ri(t) + K_b \omega(t) \dots\dots(33)$$

This sequence of equations leads to a set of two differential equations that describe the behavior of the motor, the first for the induced current,

$$\frac{di}{dt} = -\frac{R}{L} i(t) - \frac{K_b}{L}\omega(t) + \frac{1}{L} v_{app}(t) \dots\dots(34)$$

And the second for the resulting angular rate,

$$\frac{d\omega}{dt} = -\frac{1}{J}K_f\,\omega(t) + \frac{1}{J}K_m i(t) \;\ldots\ldots\ldots(35)$$

2.2) State-Space Equations for the DC Motor:

Given the two differential equations (34), (35), you can now develop a state-space representation of the DC motor as a dynamic system. The current i and the angular rate $\omega$are the two states of the system. The applied voltage,v_{app}, is the input to the system, and the angular velocity ω is the output.

$$\frac{d}{dt}\begin{bmatrix} i \\ \omega \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{K_b}{L} \\ \frac{K_m}{J} & -\frac{K_f}{J} \end{bmatrix}\begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_{app}(t) \;\ldots\ldots(36)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} i \\ \omega \end{bmatrix} + [0]v_{app}(t) \;\ldots\ldots..(37)$$

The above equations can be converted write into state space equations format.

$$\frac{dx}{dt} = Ax + Bu \;\ldots\ldots(38)$$

$$y = Cx \;\ldots\ldots(39)$$

where x is state variable, $x = (i, v)^T$, u, control input, $u = v$, and y, measurement output, $y = w$.

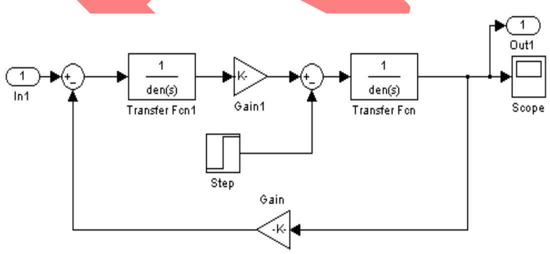
A=[-R/L -Kb/L; Km/J -Kf/J];

B=[1/L; 0];

C=[0 1];

D=[0];

Fig.2 shows the simulation diagram of simplified dc motor model. From fig.2



we know this is a second order linear system with single input and single output.

III. DESIGN OF ADAPTIVE CONTROLLER

In this section a model reference adaptive controller will be designed by using Lyapunov's stability theory, which can keep the motor dynamic performance consistent with the reference model and make the system insensitive to parameter variations and external disturbance, and the steady error goes to zero. The design steps are arranged as follows.

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First, a proper reference model is selected according to the performance index. Then the controller structure is determined and the error equation is deduced. Finally, a Lyapunov function is chosen and is used to develop parameter adaptation law, which can make the error approximate to zero.

Since the plant model has the format as eqn. (38), we assume the reference model as follows.

$$\frac{dx_m}{dt} = A_m x_m + B_m u_c \dots\dots(40)$$

Then select a control law as eqn. (41).

$$u = \theta_1 \, u_c + \; \theta_2 \, x \dots\dots(41)$$

Thus the model reference adaptive system is shown in fig.3. Now the state equation of the closed loop system has been changed to the following equation.

$$\frac{dx}{dt} = (A - B\theta_2)x + B\theta_1 u_c$$

$$\frac{dx}{dt} = A_c(\theta)x + \; B_c(\theta)u_c \dots.(42)$$

where the parameters in matrices θ_1 and θ_2 can be selected in any way, there can also exist some constraints between them.

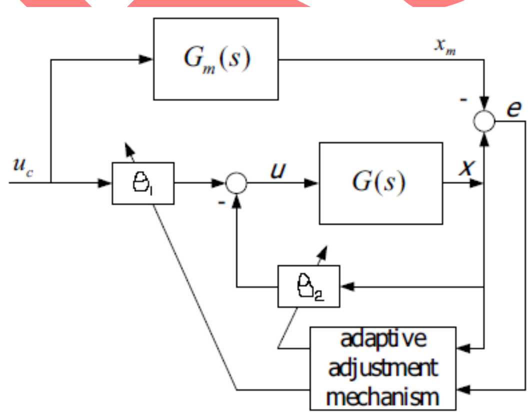

Fig. 3 Block diagram of MRAC

We suppose the closed loop system can be described with eqn. (42), where matrices A_c and B_c depend on the parameter, θ and θ is a certain combination of θ_1 and θ_2. If eqn. (42) is equivalent to eqn. (40) at any time, then the original system can follow the reference model completely. A sufficient condition is there exist a parameter θ^0 that makes eqn. (43) hold.

$$A_c(\theta^0) = A_m$$
$$B_c(\theta^0) = B_m \ldots..(43)$$

Here we introduce error *e*, which is defined in eqn.(44).

$$e = x - x_m \ldots.(44)$$

By subtracting eqn.(40) from eqn.(38), we get

$$\frac{de}{dt} = \frac{dx}{dt} - \frac{dx_m}{dt} = Ax + Bu - A_m x_m - B_m u_c \ldots.(45)$$

Adding and subtracting a term $A_m x$ at right – hand side of eqn.(45), we will get

$$\frac{de}{dt} = A_m e + (A - A_m - B\theta_2)x + (B\theta_1 - B_m)u_c$$
$$\frac{de}{dt} = A_m e + (A_c(\theta) - A_m)x + (B_c(\theta) - B_m)u_c$$
$$\frac{de}{dt} = A_m e + \varphi(\theta - \theta^0) \ldots..(46)$$

The last equality of above equation is derived when extract model tracking condition is met. To deduce the parameter tuning law, we introduce a function $V(e,\theta)$.

$$V(e,\theta) = \frac{1}{2} * \left(\gamma e^T P e + (\theta - \theta^0)^T(\theta - \theta^0)\right) \ldots..(47)$$

where *P* is a positive definite matrix. $V(e,\theta)$ is obviously a positive definite function. If its first order derivative to time is not positive definite, then *V* is a Lyapunov function. Now we solve the derivative of *V* to time *t*.

$$\frac{dV}{dt} = -\frac{\gamma}{2} e^T Q e + \gamma(\theta - \theta^0)\varphi^T P e + (\theta - \theta^0)^T \frac{d\theta}{dt}$$
$$\frac{dV}{dt} = -\frac{\gamma}{2} e^T Q e + (\theta - \theta^0)^T \left(\frac{d\theta}{dt} + \gamma\varphi^T P e\right) \ldots.(48)$$

where *Q* is a positive definite matrix, which meets the following equation.

$$A_m{}^T P + P A_m = -Q \ldots.(49)$$

According to Lyapunov's stability theory, as long as A_m is stable, there always exist such positive definite matrices *P* and *Q*.

If we choose the parameter tuning law as follows

$$\frac{d\theta}{dt} = -\gamma\varphi^T P e \ldots(50)$$

Then we will get

$$\frac{dV}{dt} = -\frac{\gamma}{2} e^T Q e \ldots.(51)$$

i.e., the derivative of Lyapunov function *V* to time *t* is half negative definite. According to Lyapunov's stability theorem, now the output error between real system and reference model will approximate to zero, and the whole system will be asymptotically stable. Therefore, eqn.(50) is the Lyapunov's stability theory-based parameter tuning law for the model reference adaptive system.

IV. SIMULATION RESULTS

The parameters of the DC motor are listed in table 1.

Armature resistance R(ohm)	7.72
Armature inductance L(H)	0.16273
Back emf constant K_b	1.25
Mechanical inertia J(Kg/m^2)	0.0236
Friction coefficient B_f (N.m./rad/sec.)	0.003
Torque constant K_t	1.25
Rated load F_L(Nw.mt)	1.2 to 2.4
Speed ω (rev/min)	1500
Power P(Kw)	0.5

In this example, we select the following reference model after several trial-and-errors.

$$G_m(s) = \frac{900}{s^2 + 47.58s + 900}$$

The reference model has small overshoot. Assume that output of reference model is y_m and input is command signal u_c.

The error expression is

$$\ddot{e} + 47.58\dot{e} + (413 + 325.5\theta_2)y - 900y_m = (325.5\theta_1 - 900)u_c$$

$$(\ddot{e} + 47.58\dot{e} - 900e) + (325.5\theta_2 - 487)y + (900 - 325.5\theta_1) = 0$$

The liapunov'sfunction is chosen as

$$v = \frac{1}{2}e^2 + \frac{1}{\gamma}[(325.5\theta_2 - 487)^2 + (900 - 325.5\theta_2)^2]$$

So the adaptive law as

$$\frac{d\theta_1}{dt} = -\frac{u_c\, e\, \gamma}{475.8 * 325.5 * 2}$$

$$\frac{d\theta_2}{dt} = \frac{y\, e\, \gamma}{475.8 * 325.5 * 2}$$

The simulation results are shown in figs.4 to 9, respectively

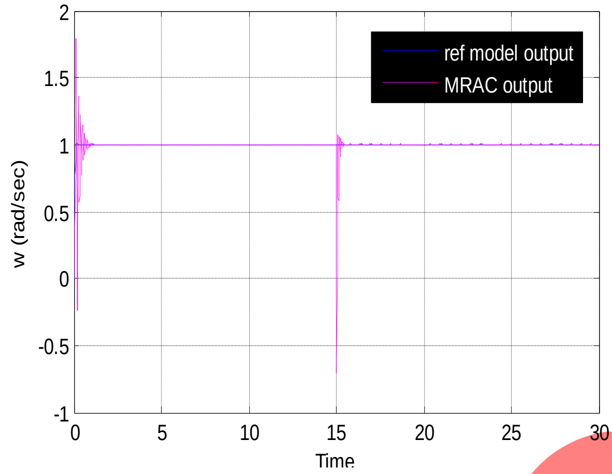

Fig.4 Step response waveform of DC motor.

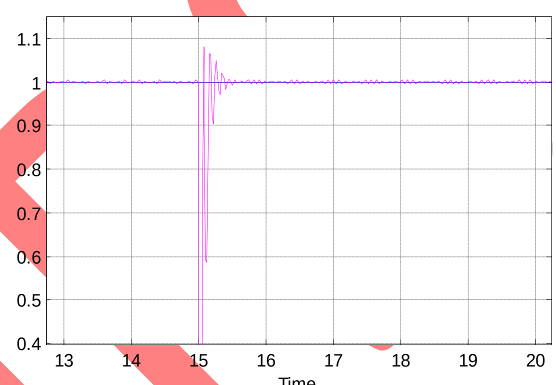

Fig.5 Zoom of the response waveform when disturbance occur at 15 sec

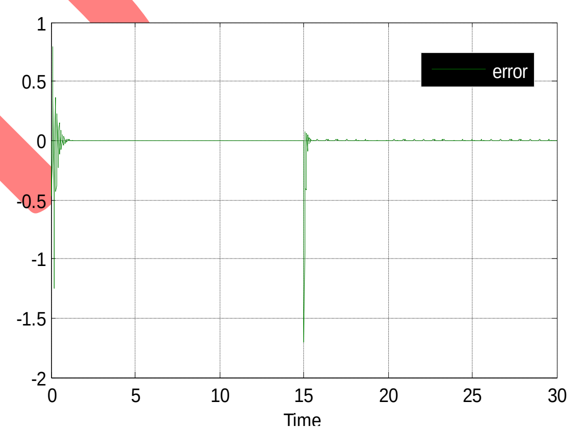

Fig.6 The waveform of output error between the MRAS and process

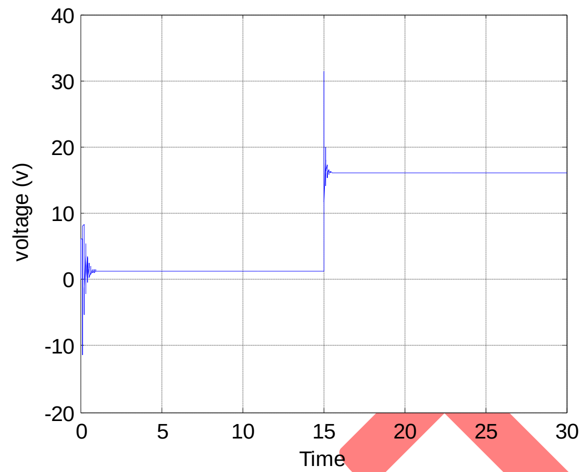

Fig.7 Control input of the MRAS

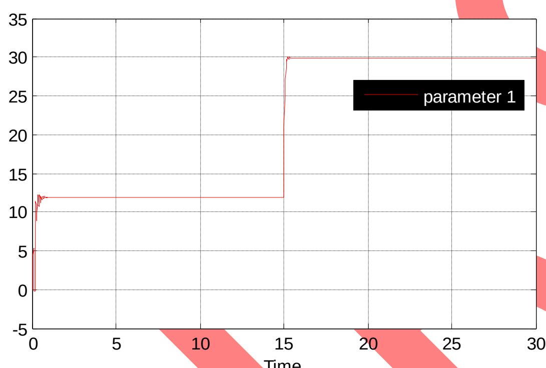

Fig.8 Tuning of the parameter θ_1 of adaptive control law.

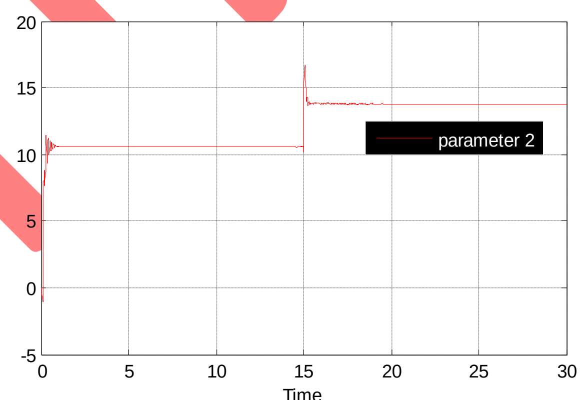

Fig.9 Tuning of the parameter θ_2 of adaptive control law.

4.2) Comments:

In simulation, the system ability of rejecting external disturbance is studied. First, when the system is under zero initial condition, the motor can follow the reference model perfectly starting from rest to steady state as in fig 4. Then, when the time is 15 sec. the load F_L suddenly changes from 0 to full load (2.5 N). At this time the model reference adaptive system suffers slight oscillation, but it can be stable very soon.Fig.5 is the zoom of response curve when the load disturbance is exerted. The MRAS has little deflection from the steady state with small magnitude of oscillation, but it is stabilized very soon. The output error between MRAS and reference model during the whole dynamic process is illustrated in fig.6. From this figure we can see the error occurs mainly at the startup stage and the settling stage while external disturbance is exerted. The error goes to zero when the system is at steady state. Fig.7 presents the input signal of MRAS, i.e., the motor input voltage. Figs.8 and 9 give the updating process of parameters θ_1 and θ_2 of the adaptive control law, respectively. The control law can automatically adjust itself when external disturbance is exerted to the system.

V. CONCLUSION

In this report simplified mathematical model of permanent magnet linear motor is developed. Then model reference adaptive controller has been designed based on the liapunov's stability theory. Simulation results shows that the liapunov's stability theory based model reference adaptive system is more robust and stable, which has better dynamic performance and stronger disturbance rejecting ability. The adaptive control law is independent of plant parameters and easy to implement. So this method is effective.

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